Issues in Tracking & Fusion

Peter Willett
University of Connecticut

NATO SET-157 Lecture Series
Multisensor Fusion:
Advanced Methodology and Applications

Outline

- Some Fusion Issues
  - fusion architectures
  - sampling times
  - sensor layout
  - use of LFM waveforms for tracking
- Target Tracking Performance Issues
  - static BCRLB with MOU
  - dynamic BCRLB with MOU
  - HYCA
- Track Testing Issues
  - SPRT
  - Page testing
  - Shiryaev testing
- As with yesterday, I truly hope that in this presentation there is some "news you can use"

FUSION 2011

Fusion 2011

Organizing Committee

General Chairs
Darin Dunham
Vectraxx, Inc., USA

Amy Sunshine Smith-Carroll
NSWC, USA

Chair Emeritus
Peter Willett
University of Connecticut, USA

Technical Co-Chairs
Lance Kaplan
Army Research Labs, USA

Neil Gordon
DSTO, AUS

Financial Chair
Juan Vasquez
Numerica Corp., USA

Tutorials Chair
W. Dale Blair
GTRI, USA

Publications Chair
Wayne Blanding
York College of Pennsylvania, USA

Special Sessions Chair
David Hall
Pennsylvania State University, USA

Awards Chair and Plenary Sessions Chair
Roy Streit
Metron, Inc., USA

Contact:
Lance Kaplan
lance.m.kaplan@us.army.mil
301.394.0807

www.fusion2011.org

14th International Conference on Information Fusion
CALL FOR PAPERS
Hyatt Regency Chicago
5-8 July 2011

Overview. The 14th International Conference on Information Fusion will be held in Chicago, Illinois USA, at the Hyatt Regency Chicago Hotel. Authors are invited to submit papers describing advances and applications in information fusion, with submission of non-traditional topics encouraged.

Conference Site. Chicago, as the third largest city in the US, offers world-class museums, dining, and entertainment while providing a small town Midwest charm with its many parks along Lake Michigan. The city is a major financial center and home to the headquarters of Boeing. O'Hare Airport serves more international cities than any other airport within the US. The Hyatt Regency Chicago is conveniently located within the heart of Chicago with easy access to the Illinois Center, Magnificent Mile, Navy Pier and many parks and museums.

All information fusion related topics are welcomed. Topics of interest include (but are not limited to):

1. **Theory and Representation:**
Probability theory; Bayesian inference; Fuzzy sets and fuzzy logic; Dempster- Shafer, evidential reasoning, brief functions, logic-based fusion and preference aggregation; Random finite set statistics; Topic modeling
2. **Algorithms:**
Registration; Detection, localization and signal processing; Automatic target recognition and classification; Nonlinear filtering, tracking and data association; Automated situation assessment, prediction, pattern and behavioral analysis; Distributed fusion; Process and sensor resource management
3. **Solution Paradigms:**
Sequential inference; Data mining; Graph Analysis; Machine learning; Ontologies
4. **Data Specific Processing and Fusion:**
Image and video; Radar; Passive sensors; Soft data sources
5. **Modeling, Simulation and Evaluation:**
Target and sensor modeling; Benchmarks; Testbeds; Fusion performance modeling and evaluation
6. **Applications:**
Aided fusion; Sensor networks; Persistent surveillance; Defense and intelligence; Security; Robotics; Transportation and logistics; Manufacturing; Economics and financial; Environmental monitoring; Medical technologies

Paper Submissions. Prospective authors are invited to submit papers electronically via the EDAS system at http://edas.info/N9229. Paper templates and submission instructions will be available both at this site and at the conference website. Paper submissions are due by 4 March 2011 and are no more than 8 pages in length. All papers must be approved for public release via the appropriate procedure of their employers/funding agencies prior to submittal.

Special Session Proposals. Proposers are invited to submit via the EDAS system (http://edas.info/N9229) the theme of the special session as well as a list of committed papers. Proposals for special sessions are due by 4 February 2011. Papers for special sessions must also be submitted for review by 4 March 2011.

Tutorial Proposals. The first day of the conference will be devoted to tutorials on information fusion. Proposals for tutorials are invited. A title and description of the tutorial and biographical sketch of the instructor are due via EDAS (http://edas.info/N9229) by 4 February 2011.

Student Paper Program. Fusion 2011 is featuring a student paper program to encourage the involvement of young engineers and scientists in information fusion. Conference fees will be discounted for all student attendees. Further details will be available at the conference website.

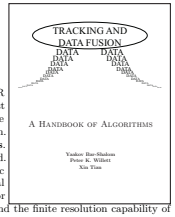
Important Deadlines

Special Session proposals due 4 February 2011	Tutorial Proposals due 4 February 2011
All papers due 4 March 2011	Notification of accepted papers 11 April 2011
Final papers due 13 May 2011	Early Registration deadline 13 May 2011

NATO OTAN

A Really Useful Book, Really

TRACKING AND DATA FUSION
A Handbook of Algorithms
Yaakov Bar-Shalom, Peter K. Willett and Xin Tian



This book, which is the revised version of the 1995 text MULTITARGET-MULTISENSOR TRACKING: PRINCIPLES AND TECHNIQUES, at double the length, is the most comprehensive state of the art compilation of **practical algorithms** for the estimation of the states of targets in surveillance systems operating in a multitarget environment using data fusion. This problem is characterized by measurement origin uncertainty, typical for **low observables**. The tools for **design** of algorithms for the association of measurements and tracking are presented. Explicit consideration is given for measurements obtained from different sensors under realistic assumptions — lack of synchronicity and different detection and accuracy characteristics. Several real-data examples are given to illustrate the techniques discussed. The modeling accounts for target maneuvers, non-unity detection probability, false alarms, interference from other targets and the finite resolution capability of sensors. The problems of track initiation, maintenance and multisensor data fusion are considered. The optimization of certain signal processing parameters based on tracking performance is also discussed. The latest results on **measurement extraction** for unresolved targets, **sensor management** and **data fusion** are included.

Many of these techniques have applications to state estimation when using multiple sensors in control systems, autonomous vehicle navigation, robotics and wireless communication. An extensive index is provided with all the indexed terms highlighted in the text for the convenience of the reader.

From a review of the 1995 book in the IEEE Aerospace and Electronic SYSTEMS Magazine:

Professor Bar-Shalom is one of the world's experts in this field... A major attraction of this book is the realistic examples, using sensors including radar, sonar, passive infrared... It is clearly written, buzzword-free and a pleasure to read... (with) humorous and insightful nuggets sprinkled throughout the text... (This) book helps balance the literature by deriving practical algorithms using mathematics that is relevant and readily accessible to engineers... Fred Daum, Raytheon Corp.

CONTENTS

INTRODUCTION. Tracking, Data Association and Fusion. Terminology. Parameter vs. State Estimation in Tracking. TRACKING FILTERS. Kalman, Extended Kalman, Unscented, Particle, Interacting Multiple Model Estimators. Performance Limits. Track Before Detect. MOTION MODELS. Linear White Noise and Coordinated Turn. MEASUREMENT MODELS. Spherical Position and Range Rate. Range and Direction Sines. Passive Sensors. Measurement Accuracies, SNR and Resolution. COORDINATE SELECTION AND CONVERSION. Polar and 3-D Measurements to Cartesian. Consistency Evaluation. Tracking with Converted Measurements vs. EKF. Coordinate Selection for Passive Sensors. EXAMPLES OF DESIGN OF AN IMM ESTIMATOR. Mode Selection for the IMM. Filter Tuning.
MULTIPLE SENSORS, CLUTTER AND MULTIPLE TARGETS. Approaches to Tracking and Data Association. ESTIMATION AND DATA ASSOCIATION IN NONLINEAR STOCHASTIC SYSTEMS. A CLUTTER MODEL. The Number and Location of False Alarms. OUT OF SEQUENCE MEASUREMENTS. Algorithms for OOSM in a KF. Incorporation of OOSM into the IMM.
ALGORITHMS FOR TRACKING A SINGLE TARGET IN CLUTTER. INTIALIZATION. Two Position, One Position and Doppler. Features. CASCADED LOGIC TRACK FORMATION IN CLUTTER. Markov Chain Model. A Design Example. TRACK MAINTENANCE WITH STANDARD FILTER. Update with a Selected Measurement. Track Monitoring and SPRT. PROBABILISTIC DATA ASSOCIATION FILTER (PDAF). Algorithm. Derivations. Nonlinear Multisensor Tracking in Clutter Example. Consistency and Robustness. Augmented PDA with Features. THE OPTIMAL BAYESIAN METHOD. Probabilities of Histories. TRACKING OF A TARGET IN THE PRESENCE OF WAKE. THE ML-PDA ESTIMATOR FOR VLO TARGETS. The Likelihood Ratio with False Measurements. Track Acceptance. The CRLB in the Presence of False Measurements. Use for Passive Acoustic/ESM Sensor and CV Target; Radar and VLO Ballistic Target.
MANEUVERING TARGET IN CLUTTER. PDAF WITH RANK FEATURE MEASUREMENT. THE MULTIPLE MODEL PDAF. Real Target Application. MULTIPLE MODEL TRACK FORMATION AND MAINTENANCE. The IMMPDAF. A Real-Data Electro-Optical Sensor Example. MULTISENSOR TRACKING OF MANEUVERING TARGET IN CLUTTER. Integration of the IMM and the Multisensor PDAF. Example with Sea Skimmers. IMM TRACKING WITH TARGET AMPLITUDE FEATURE. The IMMPDAFAI. Mixing of Models of Different Dimension. ADAPTIVE ML-PDA FOR A VLO MANEUVERING TARGET. Real Data Results.
TRACKING PERFORMANCE PREDICTION IN CLUTTER. AVERAGE TRACKING ACCURACY WITH THE PDAF. The Stochastic Riccati Equation. AVERAGE TRACKER OPERATING CHARACTERISTIC. The Average Riccati Equation A HYBRID ALGORITHM FOR PERFORMANCE PREDICTION. Track Loss and its Prediction. Detection Threshold Optimization. OPTIMAL DETECTION THRESHOLD FOR MAXIMUM INFORMATION. The FIM in Clutter. DYNAMIC CRLB FOR TRACKING A TARGET IN CLUTTER. Numerical Results.
MULTIPLE TARGETS IN CLUTTER: BAYESIAN APPROACHES. THE JOINT PROBABILISTIC DATA ASSOCIATION FILTER (JPDAF). Parametric and Nonparametric JPDA. Passive Sonar Tracking of Two Crossing Targets in Clutter. MULTIPLE HYPOTHESIS TRACKING (MHT). The HOMHT. Hypothesis Probability Evaluation. Implementation. Examples. MERGED MEASUREMENTS IN MULTIPLE TARGET TRACKING. Crossing Targets. INTEGRATED TARGET DETECTION AND TRACKING. Joint Multiobject Densities and Labeling. The Probability Hypothesis Density Filter. THE PROBABILISTIC MULTI-HYPOTHESIS TRACKER.

Fusion and Confusion

- Architecture of fusion systems
  - JDL levels
  - level 1 (Tracks): 4 configuration types
  - pre-detection fusion
  - track fusion
- To fuse or not to fuse?
  - non-uniform versus synchronized sampling
  - single-scan versus multi-scan methods
- Multi-static sonar sensor throw-down
  - diminishing returns of complementarity
  - barrier scenario
  - benefits of asymmetry
- Waveform choice
  - LFM: upsweep versus downsweep

JDL Fusion Taxonomy

The *JDL model for data fusion levels* (JDL stands for Joint Directors of Laboratories) was, according to Llinas & Hall, originally the result of a partitioning scheme based on the levels of abstraction and levels of problem-space complexity. The data fusion levels[1] were defined as:

- Level 0: Estimation of states of sub-object entities (e.g., signals, features)
- Level 1: Estimation of states of discrete physical objects (entities, e.g., vehicles)
- Level 2: Estimation of relationships among entities (e.g., aggregates, intent)
- Level 3: Estimation of impacts (e.g., consequences of activities)

[1] The original partitioning of data fusion included also Level 4, designated as "Process Refinement," with the function of planning resources to tasks. However, this was recognized as a Resource Management function and, consequently it is outside the fusion realm.

Taxonomy or Taxidermy?

Data Fusion Levels	Association	Estimation	Output
L0 Signal Assessment	Observation to Signal/Feature	Measurement (“plot”) or Feature Detection/Extraction	Estimated Signal/Feature with Confidences
L1 Object/Entity Assessment	Extracted Measurement or Feature to Object State or Sensor Entity (Local Track) to Object State	Object/Entity State Estimation	Object/Entity State Estimate with Confidences
L2 Situation Assessment	Entity to Entity or Entity to Relationship or Relationship to Relationship	Relational State Estimation	Estimated Relationships and Situation with Confidences
L3 Impact Assessment	Situation to System Courses of Action	Cost-Benefit Analysis	Estimated MOPs and MOEs with Confidences

Even More Taxonomy, For Level I

Type I configuration: Single sensor situation, which serves as a baseline.

Type II configuration: Single sensor tracking followed by *track to track association and fusion*.

Type III configuration: Measurement to measurement association across sensors with all the measurements from the same time (the sensors are assumed perfectly synchronized), i.e., *static association*, followed by *central dynamic association and tracking*.

Type IV configuration: Completely *centralized association and tracking*.

Type I Configuration

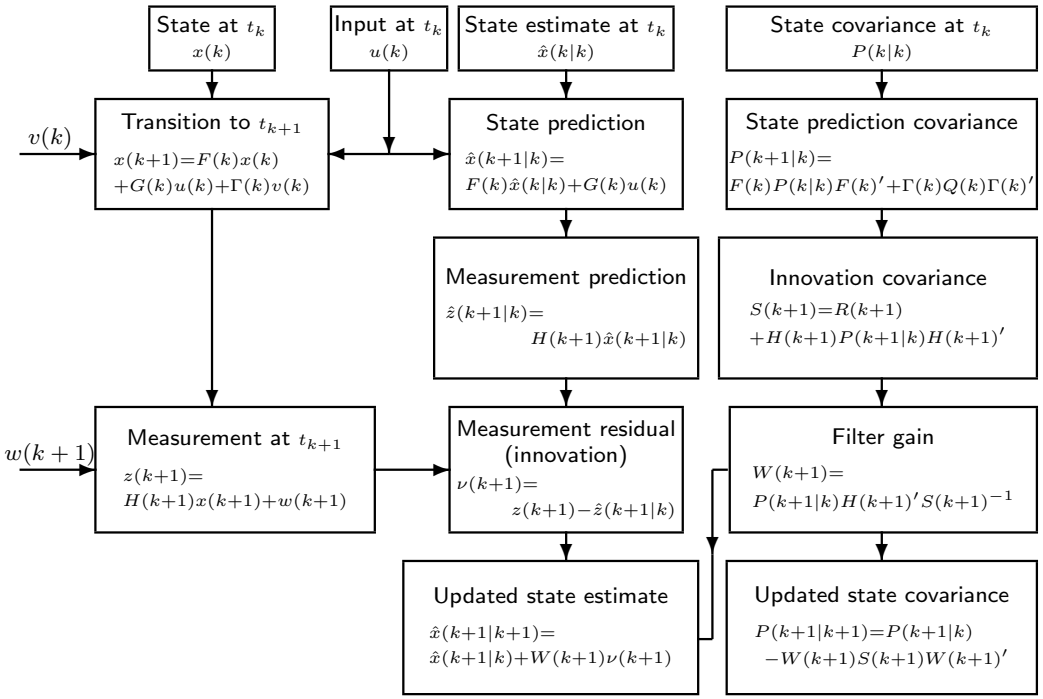
- Single sensor situation.
- In a multisensor situation this corresponds to *reporting responsibility (RR)*. Each sensor operates alone and has responsibility for a certain sector of the surveillance region — no fusion of the data (measurements or tracks) from the multiple sensors is done.
- As targets move from one sector to another, they are handed over – *handoff* – in a manner that depends on the system. Generally, the mechanism is to assign responsibility to the sensor with the highest expected accuracy, although workload and communication constraints can also play a role.

Type IV Configuration

- Completely centralized association and tracking.
- For realistic multi-sensor processing must allow for out-of-sequence measurements (OOSMs).
  - can happen because plots arrive via network, perhaps datagram routing
  - optimally: recompute entire solution when OOSM arrives – avoid this effort
  - exact single-gain “corrector” solution for single-lag case
  - approximate single-gain “corrector” solution for multi-lag case
- Sensors need not (and should not be assumed to) be synchronized.

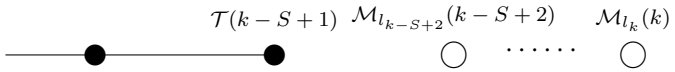
Type IV Configuration: Using the Local Tracks

Just invert the Kalman filter[2] to re-create the sensors' observations at the fusion center.

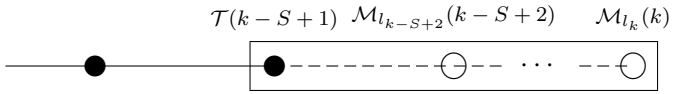


[2] This "workhorse" image was stolen shamelessly from Bar-Shalom, Kirubarajan & Li.

Type IV Configuration: Using Assignment Directly

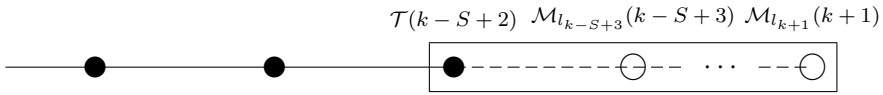


(a) Initial track sufficient statistics $\mathcal{T}(k-S+1)$ and measurement scans $\mathcal{M}_{l_{k-S+2}}(k-S+2), \ldots, \mathcal{M}_{l_k}(k)$



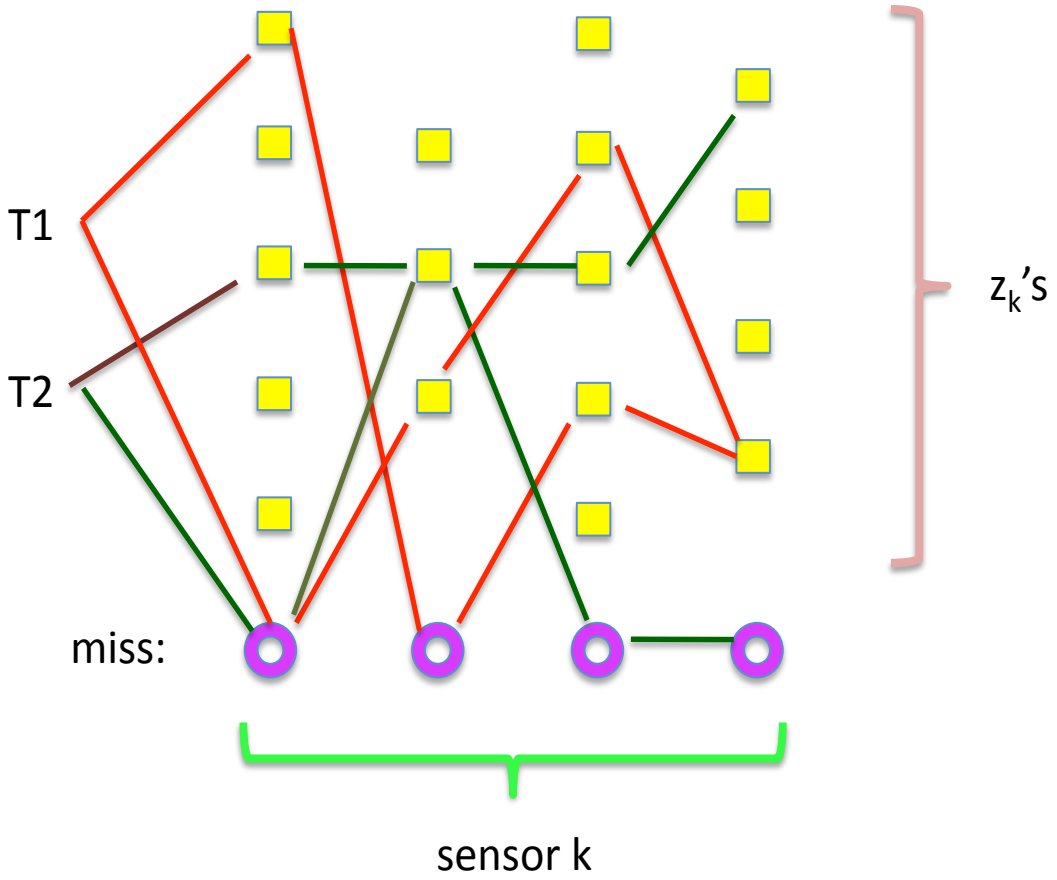
(b) S-D association between $\mathcal{T}(k-S+1)$ and $\mathcal{M}_{l_{k-S+2}}(k-S+2), \ldots, \mathcal{M}_{l_k}(k)$

(c) Updated sufficient statistics $\mathcal{T}(k-S+2)$



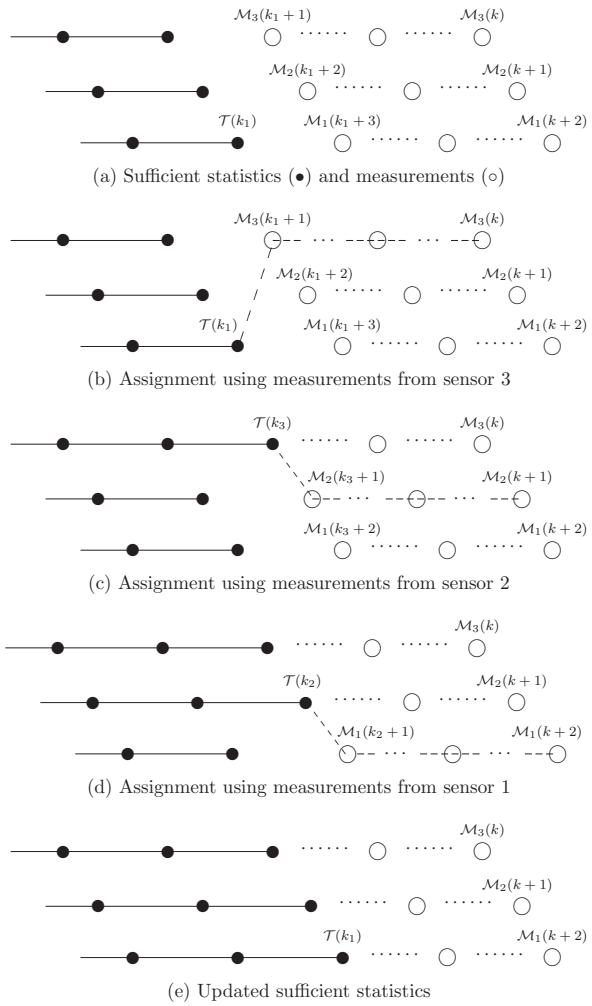
(d) S-D association between $\mathcal{T}(k-S+2)$ and $\mathcal{M}_{l_{k-S+3}}(k-S+3), \ldots, \mathcal{M}_{l_{k+1}}(k+1)$ in the next revisit

Type IV Configuration: Remember the MFA from Data Association?



- maximize LLR with constraint that no more than one contact per target per sensor
- subgradient solution

Type IV Configuration: Maximum Time Depth



(a) Sufficient statistics (•) and measurements (○)

(b) Assignment using measurements from sensor 3

(c) Assignment using measurements from sensor 2

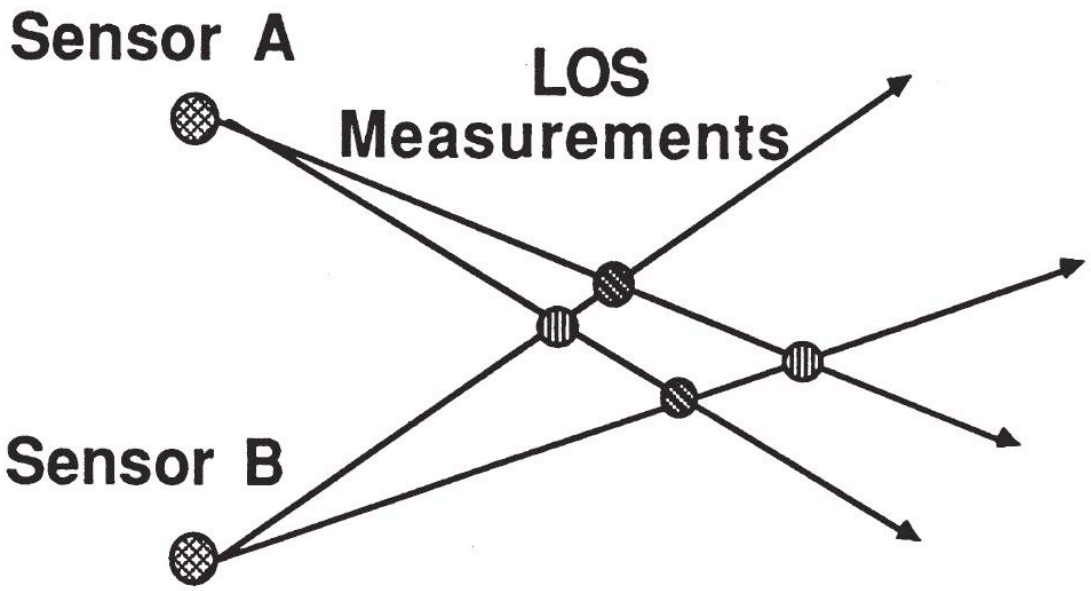
(d) Assignment using measurements from sensor 1

(e) Updated sufficient statistics

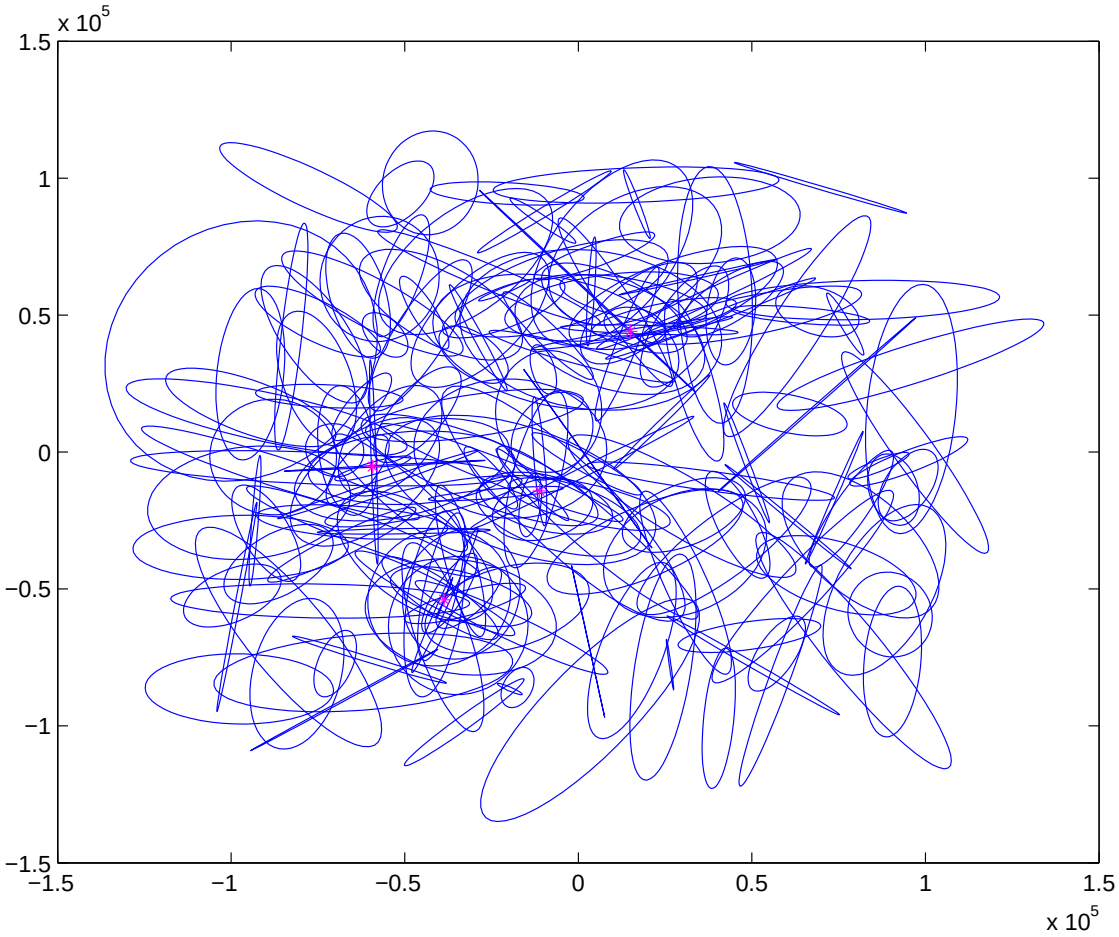
- have fixed budget of number of frames we can associate
- if too many sensors previous "direct" solution had no more time depth than GNN
- different assignment solution

Type III Configuration

- measurement to measurement association prior to tracking
- here is an example of de-ghosting for AOA sensors



Type III Configuration: Pre-Detection Fusion



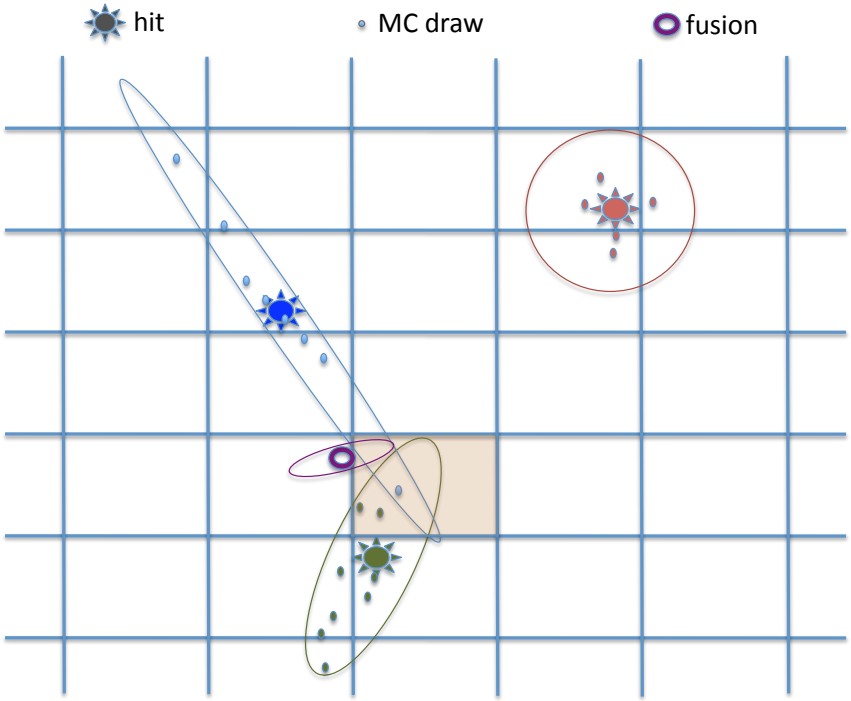
Synthetic example of detections to be fused. Covariances are random, $P_d = 50\%$, 25 sensors, $\lambda = 5$. There are four "true" targets illustrated by magenta stars.

Type III Configuration: Pre-Detection Fusion

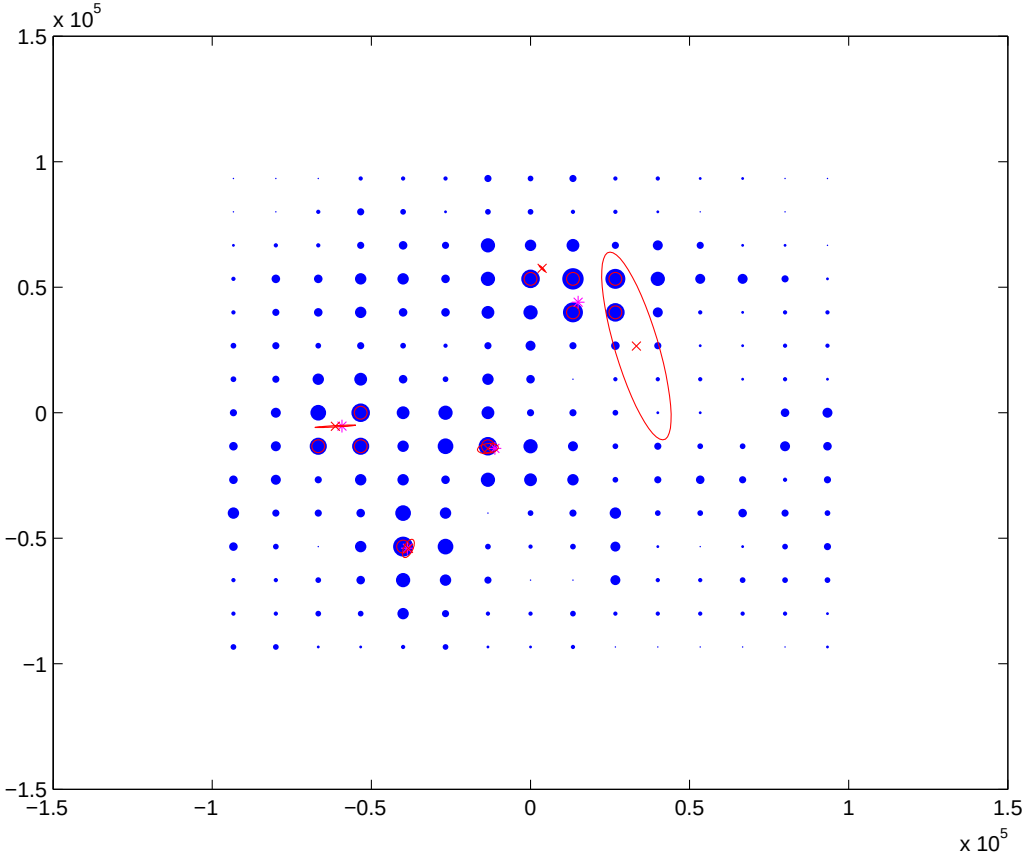
- For each contact, generate 100 samples via MC according to the contact's measurement error covariance matrix. Without this step a large covariance measurement would still only be "seen" in the grid cell at the measurement's nominal value.
- "Sift" these according to a grid (we use 25×25): If a contact yields a sample that is quantized to a grid cell, then that contact is added to the cell's list.
- Test each grid cell by number of hits, with threshold according to binomial distribution and desired fused P_{fa}.
- For cells that pass test, use PMHT measurement model and EM algorithm on that cell's listed contacts to refine estimated measurement location and estimate posterior covariance.
- Merge detections that gate with each other, since often neighboring cells have used the same detections from the initial MC step.

Cartoon

- all three contacts generate MC versions of themselves to see to which cells they pertain
- two of them are potentially associated with the shaded cell, that ultimately is basis for a fused contact
- the fused contact is based on maximizing $p(z_{blue}, z_{green}|x)$

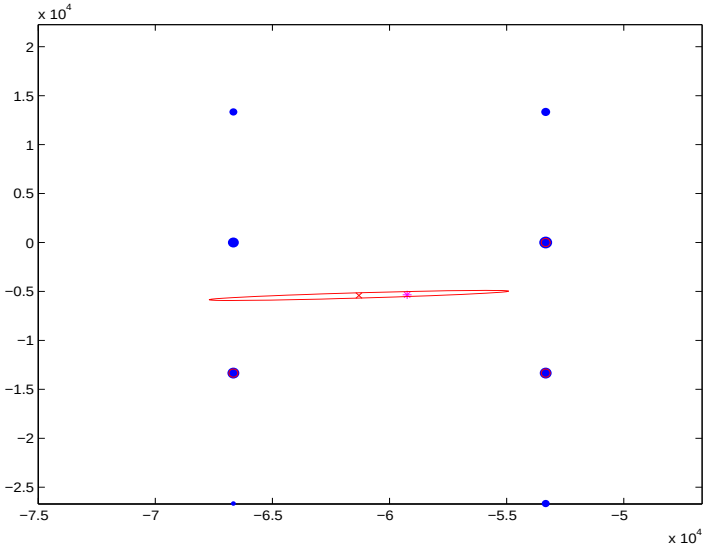


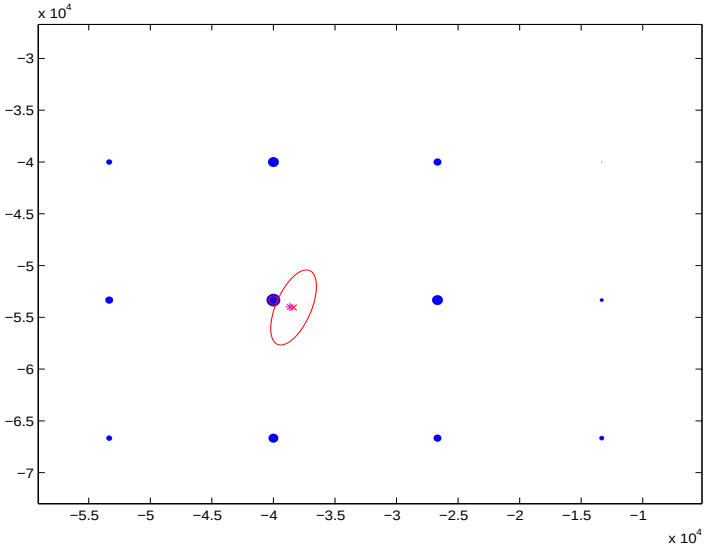
Pre-Detection Fusion: Revisit the Example

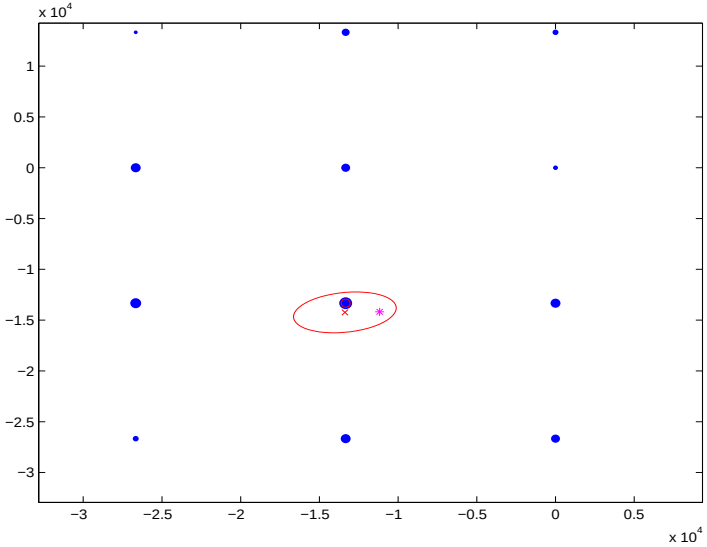


- Magenta stars are the four true contacts.
- Size of blue dot indicates number of detections in each cell's list.
- Red circle in blue dot indicates declared detection.
- Red covariance ellipses are after pre-detection fusion.
- Note miss and false alarms in upper right, others are good. Five contacts are declared rather than four.

Pre-Detection Fusion: Detailed View







Zoom view of the three correctly-fused targets from previous slide.

Pre-Detection Fusion on Scenario 1

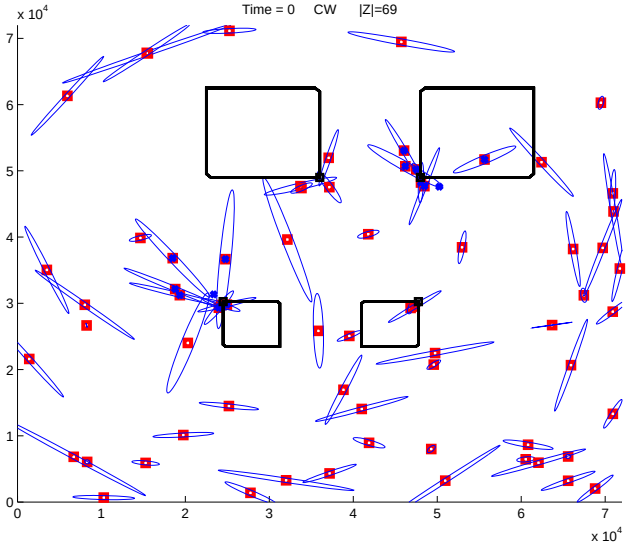


Figure: After contact winnowing based on amplitude and Doppler. Note target #3: without the MC step these measurements would not be associated together.

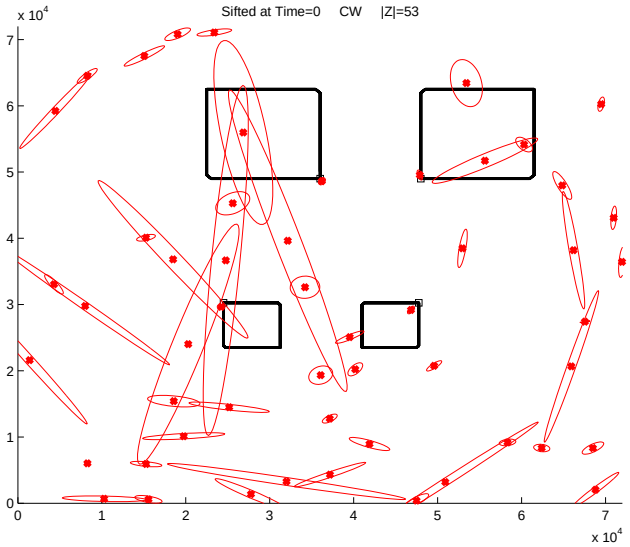


Figure: After pre-detection fusion. Although there are still many contacts, note the single low-covariance ones at SW corner of #1, SE of #2, NE of #3 & NW of #4.

Pre-Detection Fusion Comparison

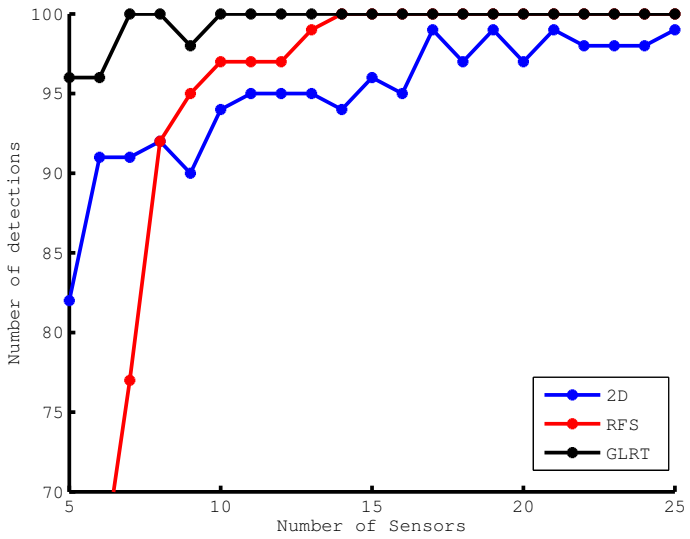


Figure: Number of successful detections of the target (out of 100 MC runs) vs. Number of sensors.

	$\lambda V = 1$	$\lambda V = 3$	$\lambda V = 5$
2D	0.054	0.099	0.105 sec
RFS MCMC	1.158	2.135	3.129 sec
GLRT	120.6	680.7	2125 sec

Table: Run time vs. Average number of false alarms per sensor.

Type II Configuration: Track Fusion

- Each sensor maintains its own (distributed) track.
- The is often the preferred solution.
- Solution is robust to failure and relatively light in its communication requirements.
- Issues:
  - sensor registration & bias
  - track-to-track association (T2TF)
  - correlation between distributed tracks?
  - fused covariance?

Type II Configuration: What is the Problem?

- $x \sim N(\bar{x}, P_x)$, $w_i \sim N(0, R_i)$ where R_i is observed by sensor i but cannot be predicted a priori

$$\begin{aligned} z_1 = x + w_1 \quad &\rightarrow \quad \hat{x}(z_1) = \bar{x} + P_x H^T (H P_x H^T + R_1)^{-1} (z_1 - H\bar{x}) \\ &\qquad\quad \hat{P}_1(R_1) = P_x - P_x H^T (H P_x H^T + R_1)^{-1} H P_x \\ z_2 = x + w_2 \quad &\rightarrow \quad \hat{x}(z_2) = \bar{x} + P_x H^T (H P_x H^T + R_2)^{-1} (z_2 - H\bar{x}) \\ &\qquad\quad \hat{P}_2(R_2) = P_x - P_x H^T (H P_x H^T + R_2)^{-1} H P_x \end{aligned}$$

  Fusion center sees $\{\hat{x}(z_i), \hat{P}_i(R_i)\}$ and can invert to $\{z_i, R_i\}$.

- But suppose

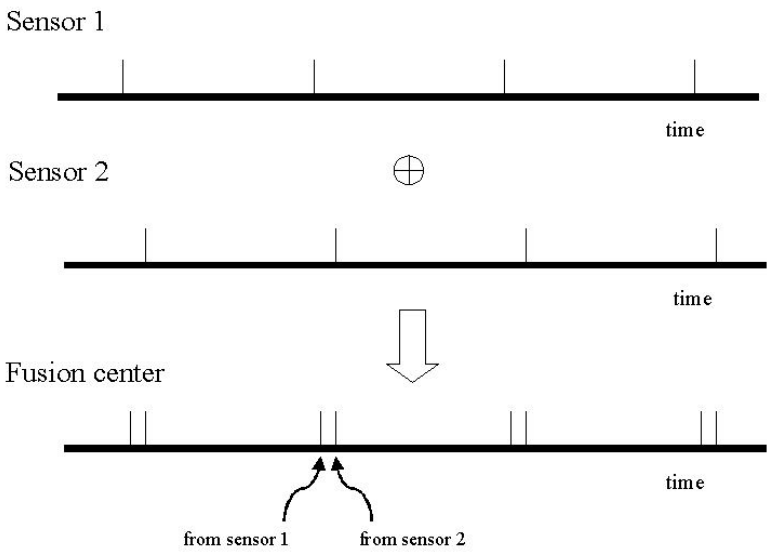
$$\begin{aligned} z_1(j) &= x(j) + w_1(j) \\ z_2(j) &= x(j) + w_2(j) \end{aligned}$$

  Fusion center sees $\{\hat{x}_i(k), \hat{P}_i(k)\}$ – it *cannot* invert to $\{z_i(j), R_i(j)\}_{j=1}^{k}$.

- $\hat{x} = \frac{1}{2} \sum_i \hat{x}_i(k)$ is <u>unbiased</u> but neither optimal nor of known quality.

Uniform vs. Nonuniform Sampling

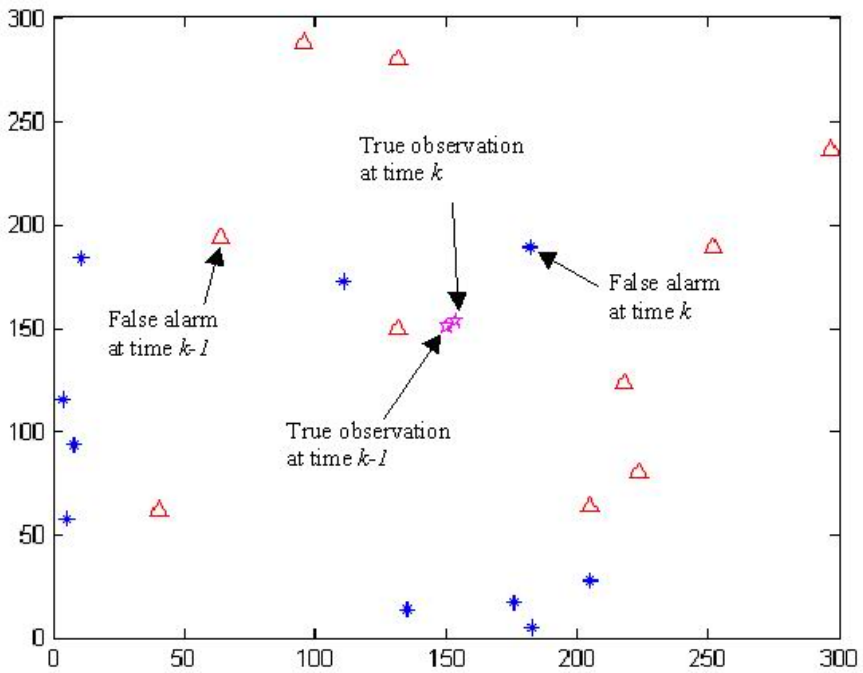
A pair of sensors could be simultaneous (fusion of detections) or else managed to have staggered sampling times.



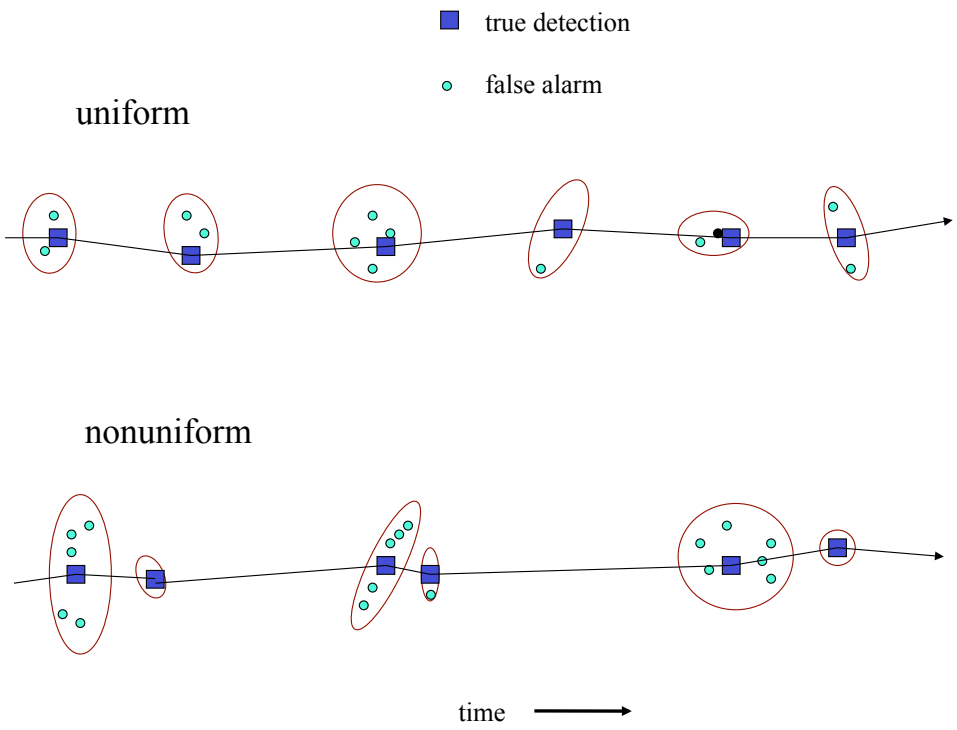
Based on Kalman filtering – that is, with no data association concerns – a uniform sampling strategy is always the best one based on a time-averaged error-covariance volume analysis.

- R. Niu, P. Varshney, K. Mehrotra and C. Mohan, "On Temporally Staggered Sensors in Multi-Sensor Target Tracking Systems," *IEEE Transactions on Aerospace and Electronic Systems*, October 2005.

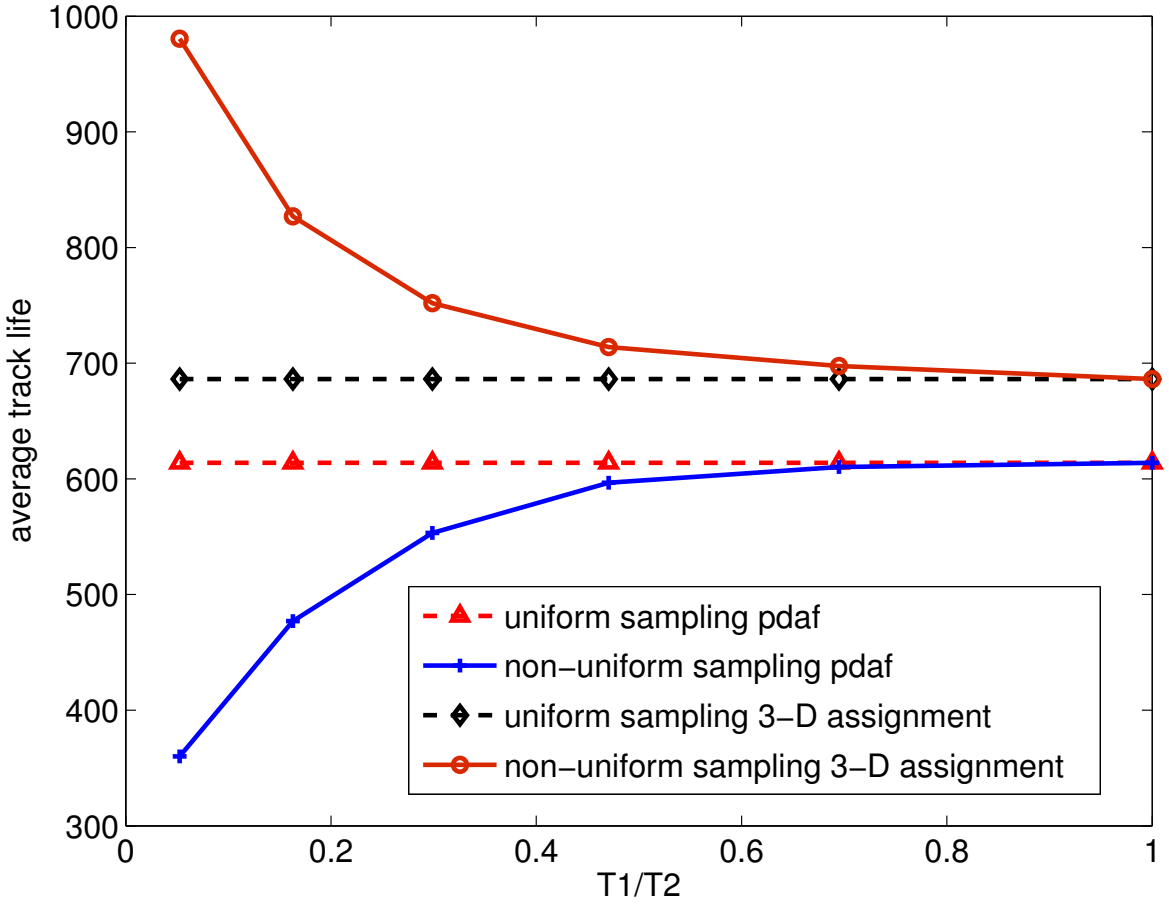
Overlay of true and false detections from two consecutive scans. False measurements are unlikely to coincide. However, given that the inter-scan time is not too great, the true measurements are close, and can corroborate one another.



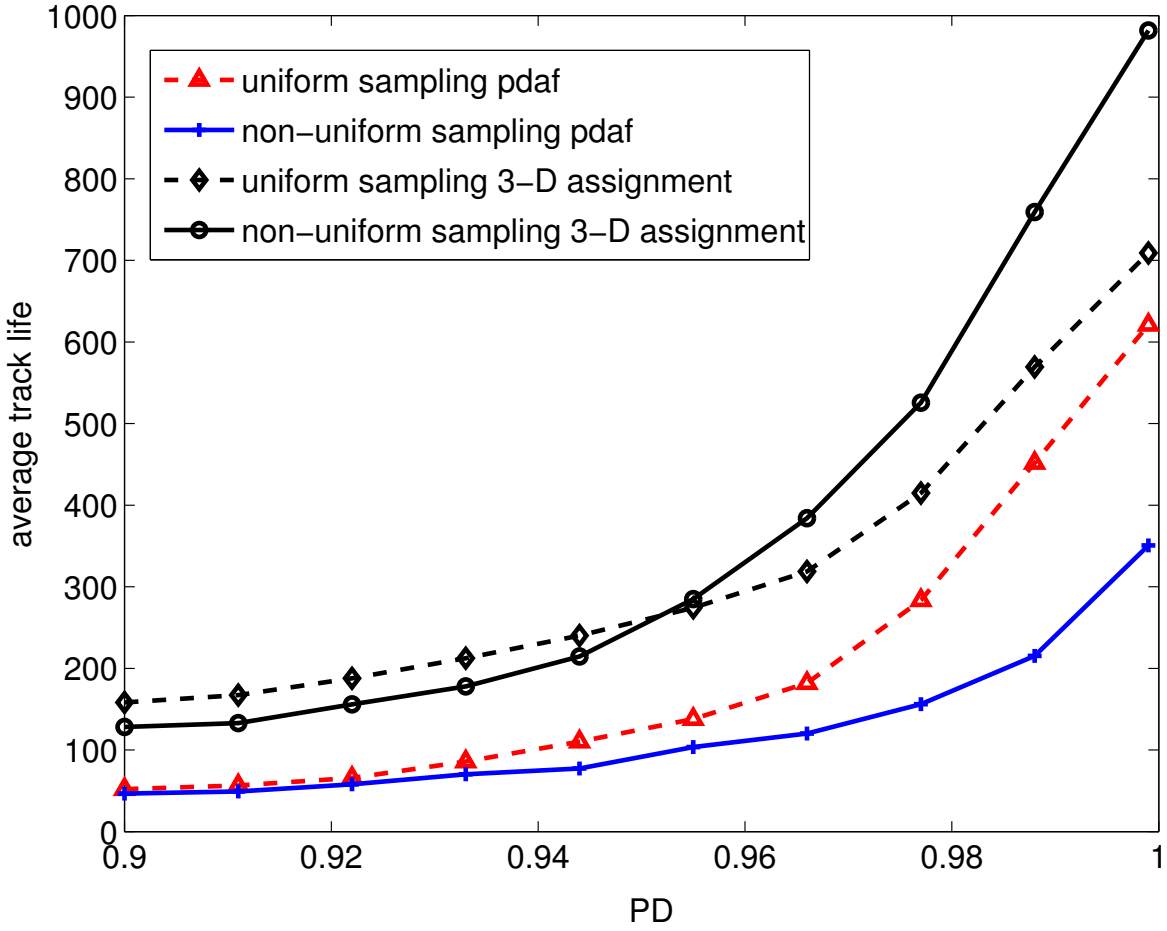
But it is not so simple. With uniform sampling the uncertainties are relatively constant in size. In the nonuniform case the ellipses increase significantly in size between pairs of samples, and allow more false alarms.



Average track life in PDAF and 3-D assignment, as a function of T_1/T_2, with uniform and nonuniform sampling schemes, false alarm density $\lambda = 0.01$, $T = 1s$, $P_D = 1$, $\tilde{q} = 10$, and $\sigma_w^2 = 0.1$.



Average track life in PDAF and 3-D assignment, as a function of probability of detection, with uniform and nonuniform sampling schemes, $T_1 = 0.1s$, $T_2 = 1.9s$, $T = 1s$, $\lambda = 0.01$, $\tilde{q} = 10$, and $\sigma_w^2 = 0.1$.



Sampling Strategy Summary

- For minimum average uncertainty using a Kalman filter there is no benefit from anything other than a uniform sampling rate.
- For a simple scan-based tracker (like the PDAF) there is no benefit from anything other than a uniform sampling rate.
- However, when the tracker utilizes more than one scan of data from the past — as with multiple-frame assignment or, presumably, the multi-hypothesis tracker (MHT) — nonuniform can outperform uniform sampling. This is particularly so when:
  - the target is highly maneuverable; and/or
  - the false alarm density is high; and/or
  - the probability of detection is high.

Multistatic Sensor Layout

We discuss optimal sensor placement for multistatic sonar systems in the LFM-only case, the CF-only case, and the combined LFM-CF case. An important aspect of the algorithm is that we employ a "minimax" criterion which results in a balanced surveillance performance. This makes sure that there is no safe path across a barrier for a target.

- aspect angle dependent target strength model.
- target Doppler is included in the localization analysis whenever CF waveforms are used.
- "blanking zone" due to direct blast signal reception.
- signal-excess is calculated by a model where a simplified reverberation-limited sonar equation.

- O. Erdinc, P. Willett and S. Coraluppi, "Multistatic Sensor Placement: A Tracking Approach," *Journal of Advances in Information Fusion*, Vol. 2, Number 1, pp. 22-34, July 2007.

- sum the information over the source and receiver locations and account for detection

$$I_{fused}(X_t) = \sum_{\forall\omega} \sum_{\forall(s,r)} P_d^{\omega}(s,r) R(X_s, X_r, X_t, \omega)^{-1}$$

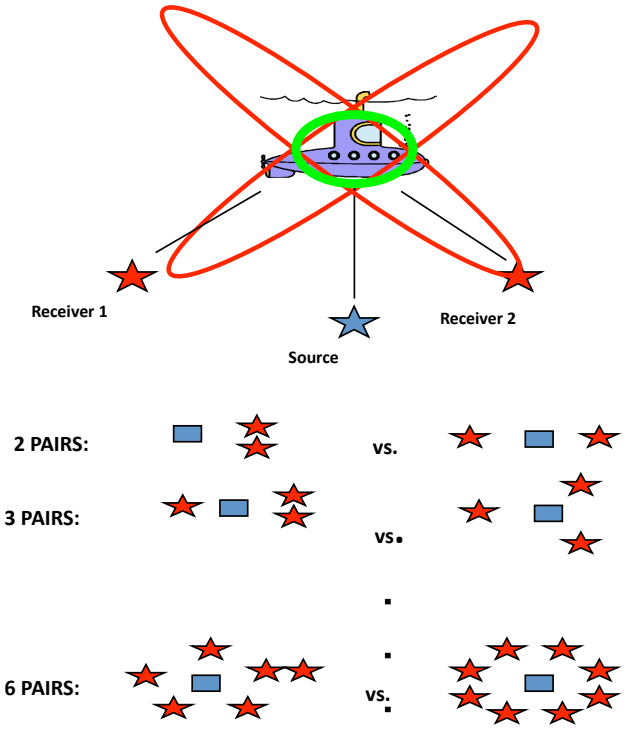
- as the scalar measure for each trajectory $T_i \in T$, we use the summation of determinants of the fused information matrix over all waypoints $w_{ij} \in T_i$:

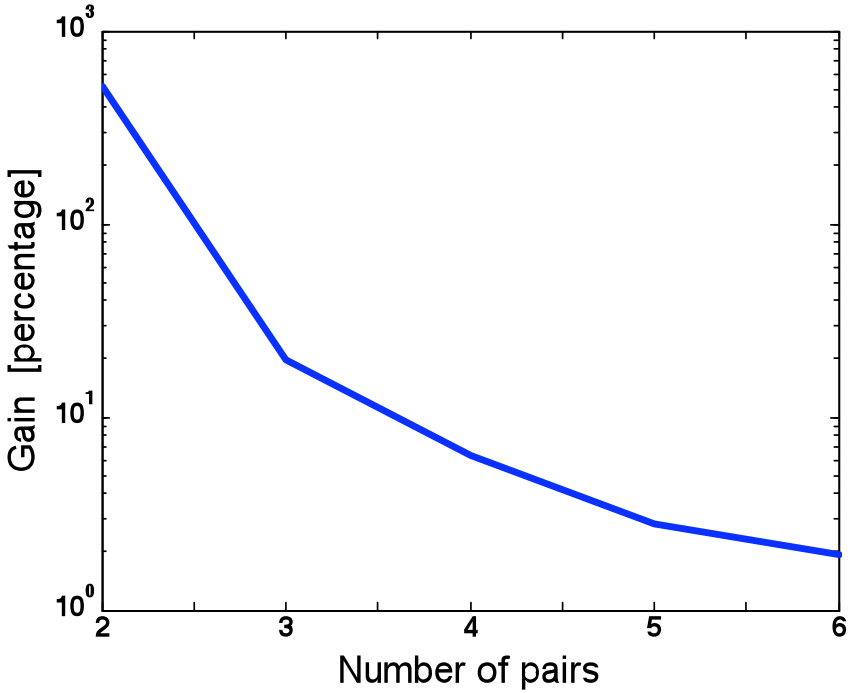
$$M(T_i) = \sum_j det(I_{fused}(w_{ij}))$$

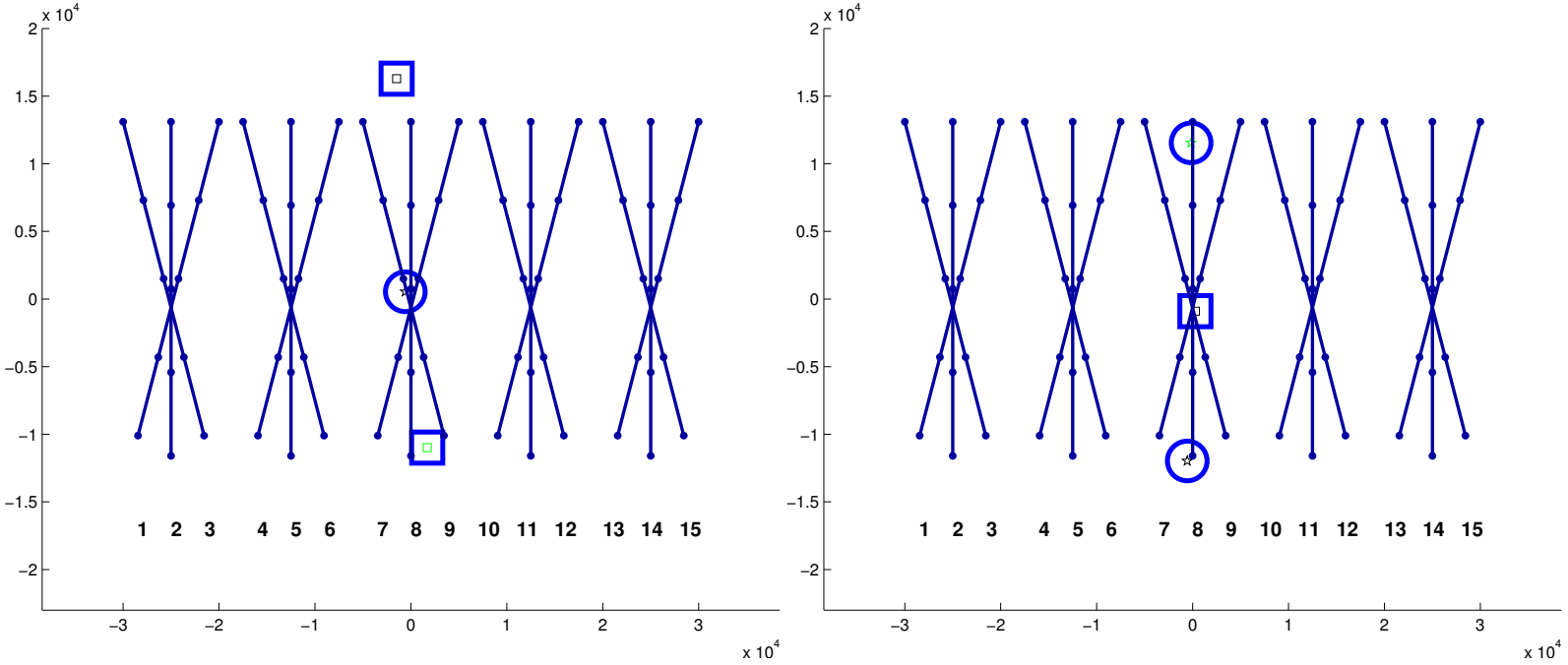
- objective function J is defined as the *worst-case* (i.e. smallest) information gain achieved across all trajectories:

$$J = \min_i M(T_i)$$

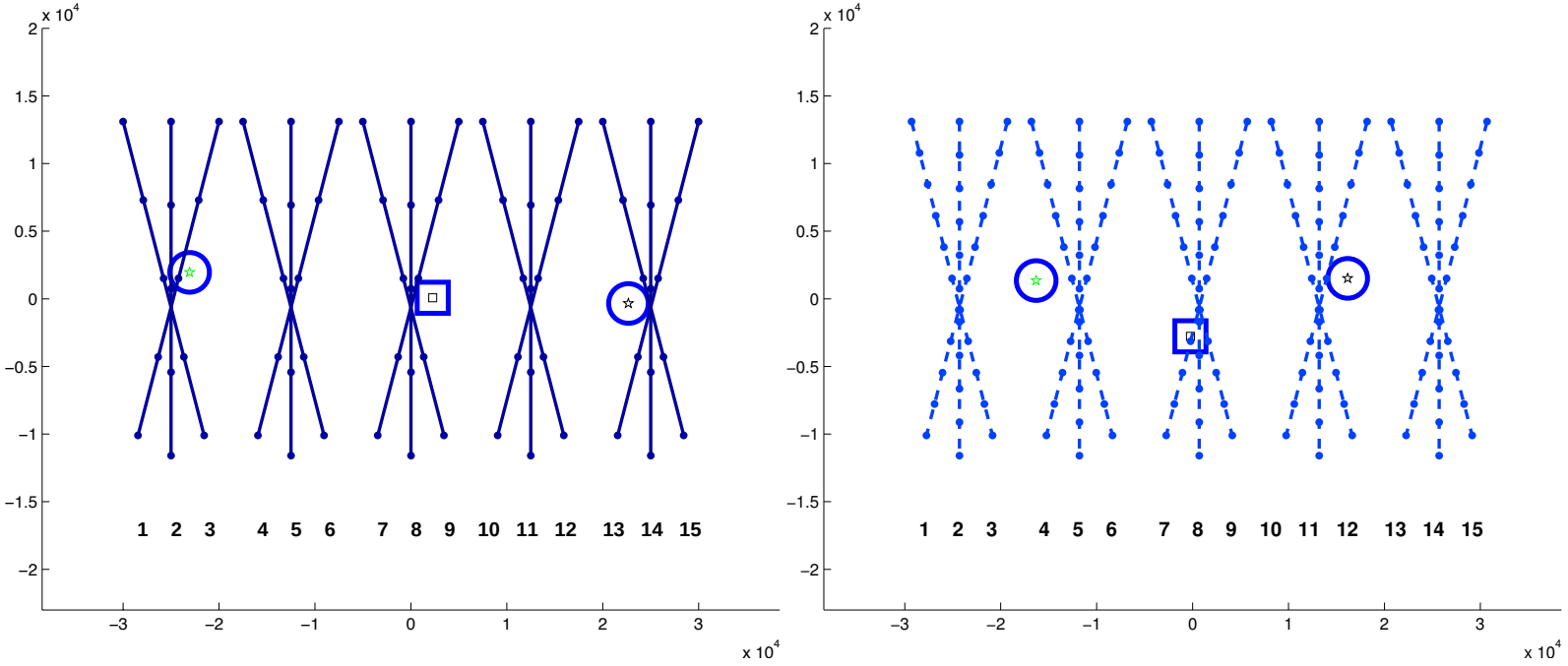
As more sensors (bistatic pairs) are added the relative gain from covariance diversity decreases markedly. The implication is that fusion is especially useful for detection of targets via detection diversity, but perhaps not from geometric diversity.







LFM waveforms. The lines are trajectories and the dots represent waypoints on each. Targets head south. The optimal placement forms a line in the North-South direction.
Left: The 2 Sources (the squares), 1 Receiver (the circle) case.
Right: The 1 Source, 2 Receivers case.



The CW case, both with one source and two receivers.
Left: Target speed is high, 10 knots.
Right: Target speed is low, 4 knots.

Sensor Layout Summary

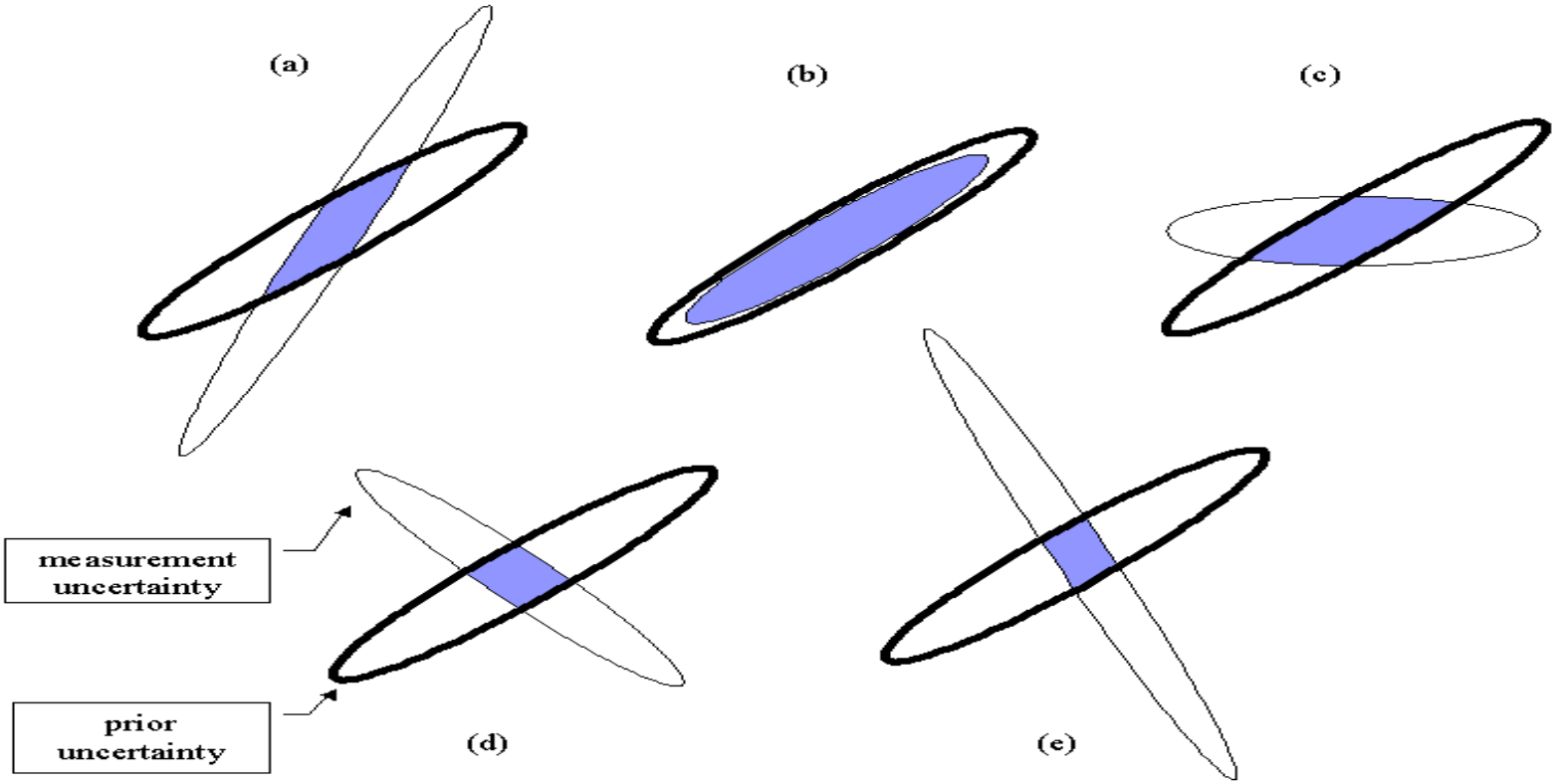
- simple & quick deployable methodology for multi-static sensor layout based on tracking performance
- saturation of benefits of fusion based on measurement covariance complementarity after bistatic pairs
- LFM waveforms: layout along barrier to see good aspect angles on all trajectories
- CW waveforms: layout across barrier to see good Dopplers
- results are robust to perturbed trajectories
- for example, the slight asymmetry persists

LFM Waveforms for Tracking

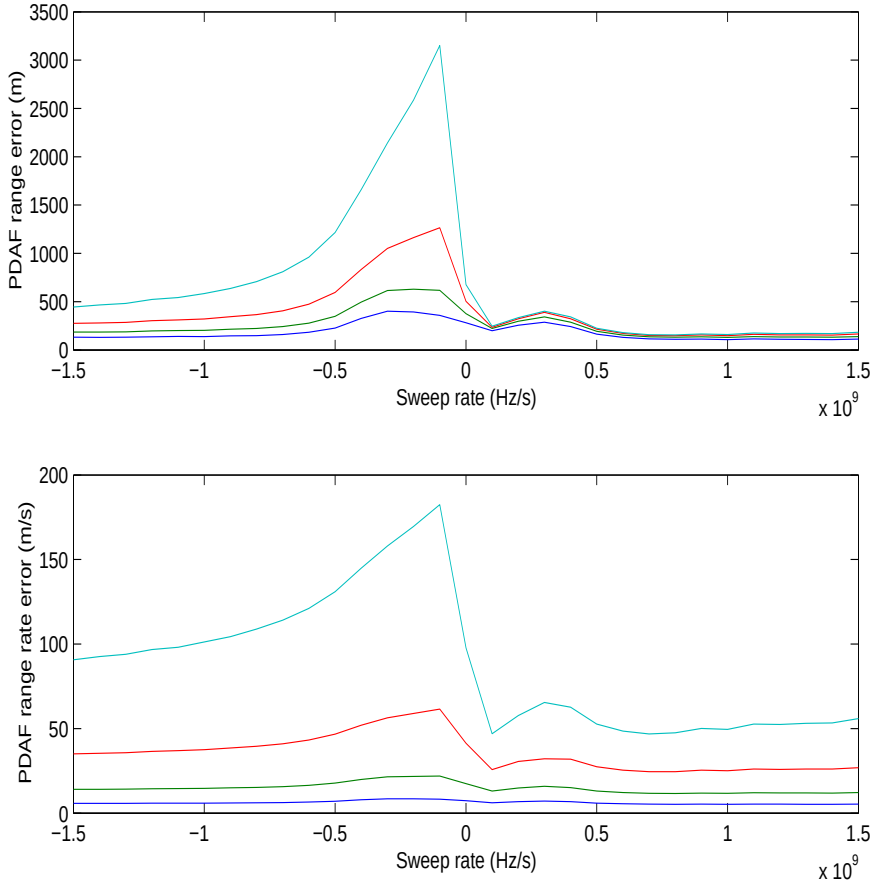
- it can be shown that a kinematic (i.e., "$F = ma$") target has positive correlation between its errors in range and in range-rate – that is, the off-diagonal terms of $P(k|k-1)$
- it can be shown that an LFM upsweep waveform has an ambiguity function that is positively correlated between range and range-rate, and tends to have contours that match an ellipse fairly well
- an LFM upsweep waveform sampled in range and range-rate is well-modeled as having Gaussian noise with correlation proportional to the negative of the sweep rate

- R. Fitzgerald, "Effects of Range-Doppler Coupling on Chirp Radar Tracking Accuracy," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. AES-10, pp. 528–532, July 1974.

Illustration of the complementarity of prior (heavy-shaded) and measurement (light-shaded) uncertainties for LFM waveforms, in range (horizontal) and range-rate (vertical) coordinates. The blue shaded region is a cartoon of the posterior uncertainty.



Estimation range and range-rate errors vs. sweep rate for a single LFM pulse. Radar parameters: $t_p = 100\mu s$, actual target velocity is $400\ m/s$. From bottom to top, the lines correspond to process noise levels ($\sigma_v = 1, 10, 100, 1000$).



LFM Waveform Summary

- LFM upsweep uncertainty is complementary to prior tracking uncertainty, and errors are well-modeled as Gaussian
- non-LFM waveforms often have “busier” ambiguity function, and are less well-modeled as Gaussian
- with Doppler-dependent clutter the situation is more complicated, but the conclusion is the same
- can also explore in case that only range measurements are available
  - then the range/range-rate coupling results in a positive bias
  - this bias can be canceled by averaging upsweep and downsweep LFMs
  - Fitzgerald originally showed that this might be counter-productive via a Kalman analysis, better to use all-upsweep.

Leaps and Bounds

- background
  - the CRLB
  - the PCRLB (or BCRLB)
- static cases
  - measurement origin uncertainty (MOU)
  - information reduction factor (IRF)
  - the case of simple scaling
- dynamic cases
  - Tichavsky, Muravchik & Nehorai's general dynamic BCRLB
  - the Gaussian case
  - the BCRLB for tracking in clutter
- other approaches for the dynamic case
  - measure sequence conditioned BCRLB
  - HYCA

The CRLB

Under (fairly) broad regularity conditions:

Theorem

For any estimator $\hat{\mathbf{x}}$ of parameter $\mathbf{x}$ using data $\mathbf{Z}$ satisfies

$$\mathbf{E}\left\{[\hat{\mathbf{x}}(z)-\mathbf{x}]\,[\hat{\mathbf{x}}(z)-\mathbf{x}]^T\right\} \;\geq\; \mathbf{J}^{-1}$$

in which

$$\mathbf{J} \;\equiv\; \mathbf{E}\left\{[\nabla_{\mathbf{x}}\log\left(p(z;\mathbf{x})\right)]\,[\nabla_{\mathbf{x}}\log\left(p(z;\mathbf{x})\right)]^T\right\}$$

is Fisher's information matrix.

Again under broad regularity conditions, if an MLE for $\mathbf{x}$ exists, then it achieves the CRLB asymptotically.

The BCRLB

The *Bayesian* or *posterior* CRLB (BCRLB or PCRLB) applies when the parameter being estimated is a random variable, as opposed to a parameter.
Let $\hat{x}(z)$ be an *unbiased* estimator of x. The CRLB on the estimation error is

$$E\{[\hat{x}(z) - x][\hat{x}(z) - x]^T\} \geq J^{-1}$$

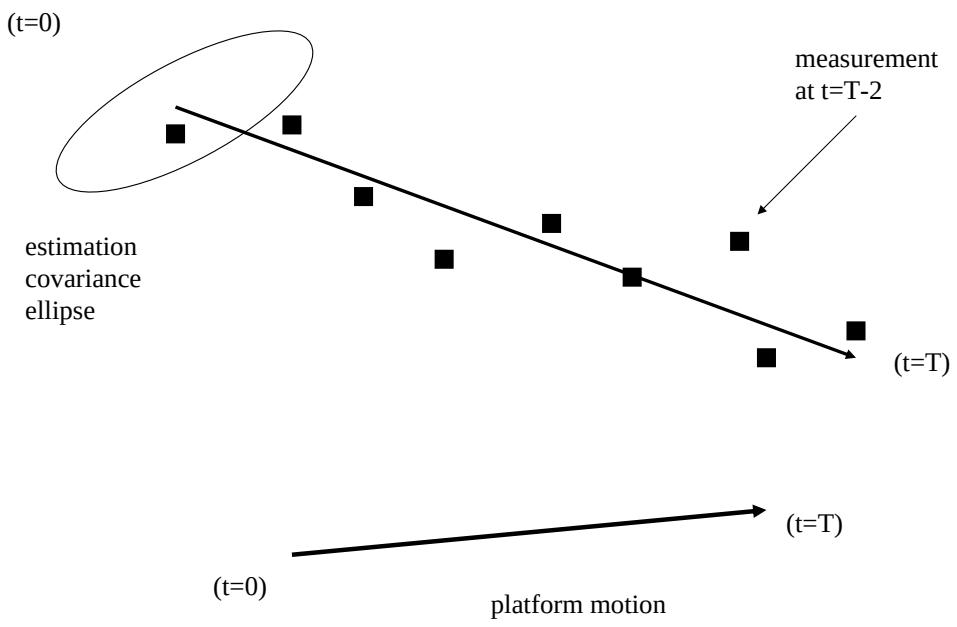
in which E denotes expectation over (z, x), and

$$\begin{aligned} J &\triangleq E\{[\nabla_x \ln(p(z,x))][\nabla_x \ln(p(z,x))]^T\} \\ &= -E\{\nabla_{xx} \ln(p(z,x))\} \end{aligned}$$

In general the BCRLB is achieved only by the MAP estimator, and iff $p(x|z)$ is Gaussian.

Notional CRLB for TMA

Target motion analysis: A straight line target has "clean" observations and is parameterized by its initial position and velocity (that is: a 4-dimensional vector).



CRLB for TMA with MOU

What happens when the measurements are uncertain? Then

$$p(\mathbf{Z};\mathbf{x}) = \prod_{t=1}^{T}\left(\sum_{m_t=0}^{\infty} q(m_t)\left[\prod_{i=1}^{m_t} p_0(\mathbf{z}_i(t))\right] \times \left[(1-\epsilon(m_t)) + \frac{\epsilon(m_t)}{m_t}\sum_{i=1}^{m_t}\frac{p_1(\mathbf{z}_i(t)-\mu_t(\mathbf{x}))}{p_0(\mathbf{z}_i(t))}\right]\right)$$

where

m_t = the number of measurements at time t, with pmf $q(\cdot)$

$q(m_t)$ = the probability of m_t clutter

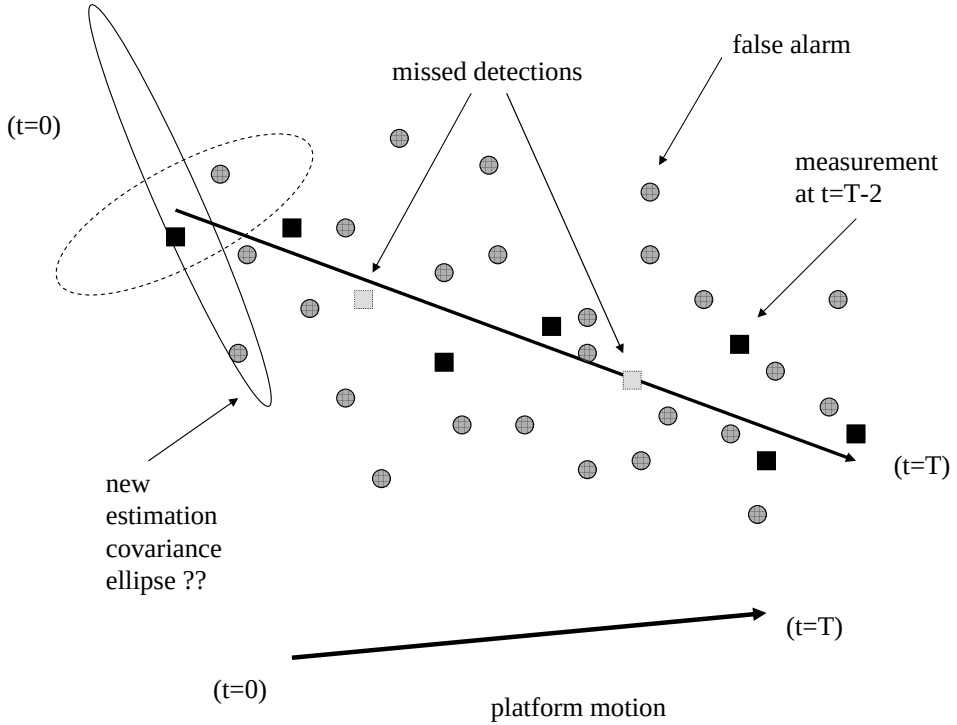
$\epsilon(\cdot)$ = the probability that a given measurement is valid

$p_0(\cdot)$ = the pdf of a false-alarm

$p_1(\cdot)$ = the pdf of a true measurement's noise

CRLB for TMA with MOU

Is this what happens?



Case: True pdf is Gaussian

Theorem

For the case that $p_1(\cdot)$ *is Gaussian we get the* **remarkable***:*

$$\mathbf{J} = \sum_{t=1}^{T} \mathbf{J}_t = q_2(P_d, \lambda V) \sum_{t=1}^{T} \mathbf{J}_t^0$$

in which

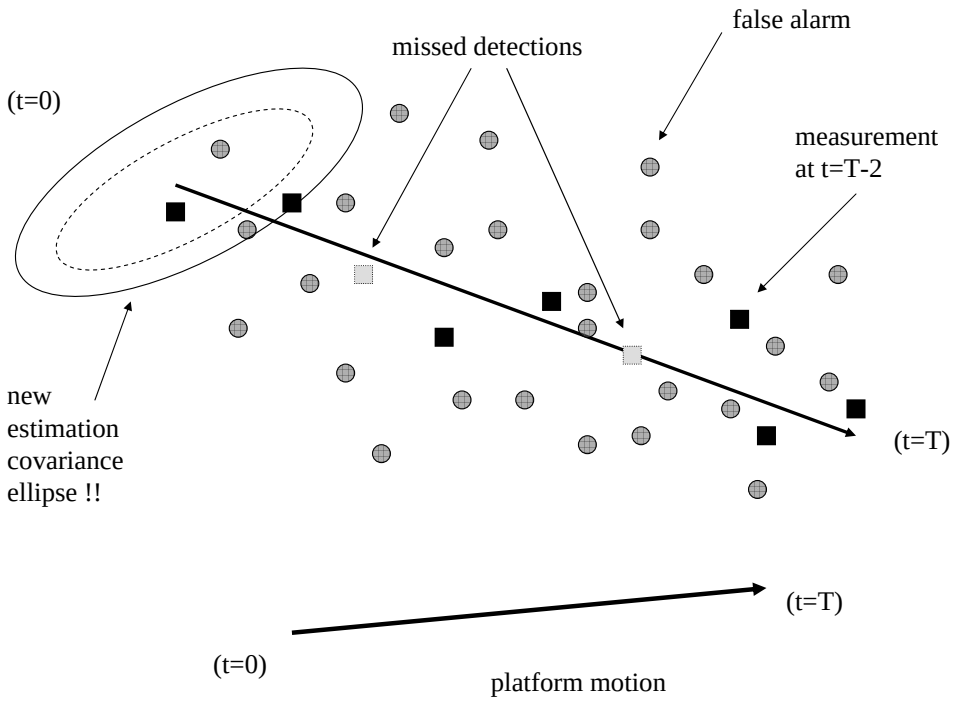
$$\mathbf{J}_t^0 \equiv \mathbf{E}\left\{ \left(\nabla_{\mathbf{x}} \log\left[p_1(\mathbf{z}_i(t) - \mu_t(\mathbf{x}))\right]\right) \left(\nabla_{\mathbf{x}} \log\left[p_1(\mathbf{z}_i(t) - \mu_t(\mathbf{x}))\right]\right)^T \right\}$$

and q_2 *is an* information-reduction factor*.*

- T. Kirubarajan and Y. Bar-Shalom, "Low-Observable Target Motion Analysis Using Amplitude Information", *IEEE Transactions on Aerospace and Electronic Systems*, pp1367-1384, October 1996.

IRF Scales CRLB

So in the Gaussian case we get this, instead. How general is this result? It is easy to see that there are some situations in which the CRLB gets “distorted” by uncertain measurements, particularly if $p_0(\cdot)$ is chosen with malice.



Some Math That You Won't Like

Define:

$$\mathbf{M}_t \equiv \begin{pmatrix} \frac{\partial \mu_t(\mathbf{x})_1}{\partial \mathbf{x}_1} & \frac{\partial \mu_t(\mathbf{x})_1}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mu_t(\mathbf{x})_1}{\partial \mathbf{x}_{n_x}} \\ \frac{\partial \mu_t(\mathbf{x})_2}{\partial \mathbf{x}_1} & \frac{\partial \mu_t(\mathbf{x})_2}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mu_t(\mathbf{x})_2}{\partial \mathbf{x}_{n_x}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \mu_t(\mathbf{x})_{n_z}}{\partial \mathbf{x}_1} & \frac{\partial \mu_t(\mathbf{x})_{n_z}}{\partial \mathbf{x}_2} & \cdots & \frac{\partial \mu_t(\mathbf{x})_{n_z}}{\partial \mathbf{x}_{n_x}} \end{pmatrix}$$

$$\mathbf{F}_t \equiv \sum_{m_t=1}^{\infty} q(m_t)\mathbf{E}\left\{ \left(\frac{\left[\frac{\epsilon(m_t)}{m_t} \sum_{i=1}^{m_t} \frac{\nabla_{\mathbf{z}}\left(p_1(\mathbf{z}_i(t)-\mu_t(\mathbf{x}))\right)}{p_0(\mathbf{z}_i(t))} \right]}{\left[(1-\epsilon(m_t)) + \frac{\epsilon(m_t)}{m_t} \sum_{i=1}^{m_t} \frac{p_1(\mathbf{z}_i(t)-\mu_t(\mathbf{x}))}{p_0(\mathbf{z}_i(t))} \right]} \right) \left(\cdot \right)^T \right\}$$

Then:

$$\mathbf{J} = \sum_{t=1}^{T} \mathbf{M}_t^T \mathbf{F}_t \mathbf{M}_t$$

A Result You Will Like

Conditions for IRF to Scale CRLB:

Theorem

C1– *If $n_z = 1$ then $\mathbf{F}_t(m)$ is a multiple of $\mathbf{F}^0$ whose value depends only on m. This is true regardless of the pdf $p_1(\cdot)$.*

C2– *If all elements of $\mathbf{z}_i$ are independent and identically-distributed (iid), and provided this distribution is symmetric, then true.*

C3– *If all elements of $\mathbf{z}_i$ can be made to be iid and symmetric via a linear transformation, then true.*

What is the IRF?

You are not going to like this.

$$q_2 = \frac{\sum_{m=1}^{\infty}\left[p(m)\epsilon(m)\int\left(\frac{\frac{\partial p_1(z_{11})}{\partial z_{11}}}{p_1(z_{11})}\right)^2 P(\chi_1|z_{11},m)\,p_1(z_{11})\,dz_{11}\right]}{\int\left(\frac{\frac{\partial p_1(z_1)}{\partial z_1}}{p_1(z_1)}\right)^2 p_1(z_1)\,dz_1}$$

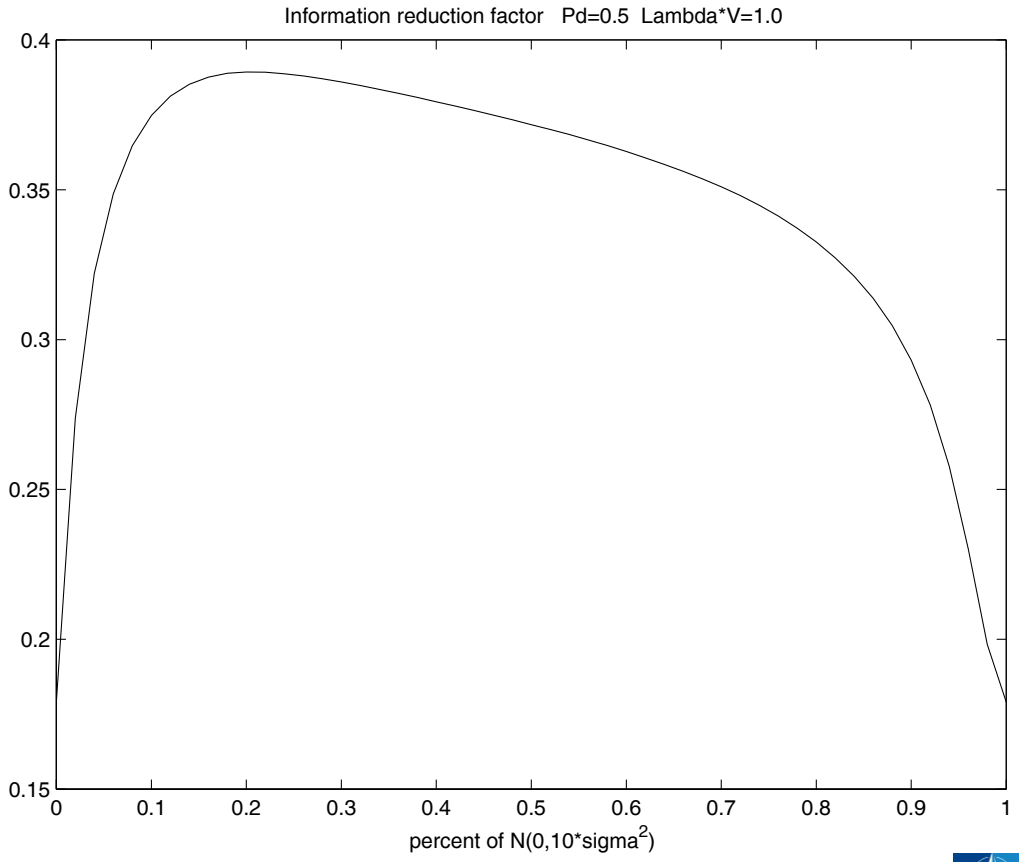
where

$$P(\chi_1 \mid z_{11}, m) \equiv \int_{\mathbf{z}_2\in V}\cdots\int_{\mathbf{z}_m\in V}\int_{\{z_{12}\ldots z_{1n_z}\}:\mathbf{z}_1\in V} \times \frac{\frac{1}{V^{m-1}}p_1(z_{12},\ldots,z_{1n_z})\left[\frac{\epsilon(m)}{m}Vp_1(\mathbf{z}_1)\right]}{\left[(1-\epsilon(m))+\frac{\epsilon(m)}{m}\sum_{j=1}^{m}Vp_1(\mathbf{z}_j)\right]}\,d\mathbf{z}_2\ldots d\mathbf{z}_m dz_{12}\ldots dz_{1n_z}$$

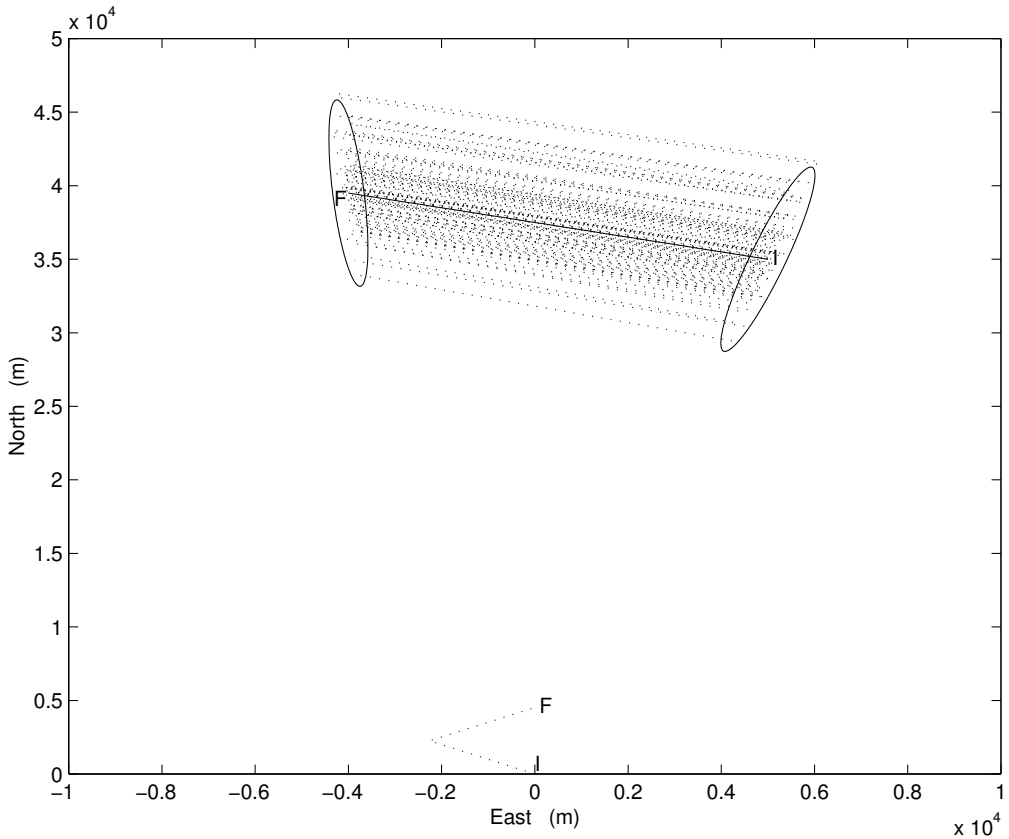
There is no reasonable way to get this other than to use Monte Carlo integration. It isn't so much work, except for the computer.

IRF Example

Information reduction factor with Gaussian mixture noise, $P_d = .5$, heavy clutter.



Deterministic-Motion Target Tracking Example



The true and estimated trajectories from 100 Monte Carlo runs for Johnson noise case, $P_d = 0.9$, $\lambda V_g = 0.1$ and kurtosis = 11.5. Note that in almost all cases the estimated values lie within their respective ellipses.

General Filtering Problem

Consider estimation in a nonlinear dynamic system

$$x_{n+1} = f_n(x_n, v_n)$$
$$z_n = h_n(x_n, w_n)$$

where x_n is the system state at time n, $\{z_n\}$ is the measurement process, $\{w_n\}$ and $\{v_n\}$ are independent white processes, and f_n and h_n are (in general) nonlinear functions. Note that this problem has noisy dynamics, noisy measurements, but no measurement origin uncertainty.

- P. Tichavský, C. Muravchik, A. Nehorai, "Posterior Cramér-Rao Bounds for Discrete-Time Nonlinear Filtering", *IEEE Trans. Signal Processing*, vol. 46, No. 5, pp. 1386–1396, May 1998.

General Filtering Result

Theorem

The sequence $\{J_n\}$ obeys the recursion:

$$J_{n+1} = D_n^{22} - D_n^{21}(J_n + D_n^{11})^{-1} D_n^{12}$$

where

$$\begin{aligned}
D_n^{11} &= E\{-\nabla_{x_n} \nabla_{x_n}^T \ln\ p(x_{n+1}|x_n)\} \\
D_n^{12} &= E\{-\nabla_{x_{n+1}} \nabla_{x_n}^T \ln\ p(x_{n+1}|x_n)\} \\
D_n^{21} &= E\{-\nabla_{x_n} \nabla_{x_{n+1}}^T \ln\ p(x_{n+1}|x_n)\} = [D_n^{12}]^T \\
D_n^{22} &= E\{-\nabla_{x_{n+1}} \nabla_{x_{n+1}}^T \ln\{p(x_{n+1}|x_n)\} \\
&\qquad + E\{-\nabla_{x_{n+1}} \nabla_{x_{n+1}}^T \ln\ p(z_{n+1}|x_{n+1})\}
\end{aligned}$$

The Gaussian Case

Using the standard F, Q, H, R notation we have

$$E\{-\nabla_{x_{n+1}}\nabla^T_{x_{n+1}}\ln\{p(x_{n+1}|x_n)\} = Q^{-1}$$
$$E\{-\nabla_{x_{n+1}}\nabla^T_{x_{n+1}}\ln\ p(z_{n+1}|x_{n+1})\} = H^T R^{-1} H$$

and so on, consequently

$$J_{n+1} = D_n^{22} - D_n^{21}(J_n + D_n^{11})^{-1} D_n^{12}$$

becomes the “information filter” form of the Riccati equation:

$$J_{n+1} = Q^{-1} + H^T R^{-1} H - Q^{-1} F (J_n + F^T Q^{-1} F)^{-1} F^T Q^{-1}$$

Special Case of General Filtering Result: MOU

The only part of the Tichavsky formula that is affected by MOU is the second term of D_n^{22}:

$$\begin{aligned} J_Z &\triangleq E\{-\nabla_{x(k)}\nabla_{x(k)}^T \ln\ p(Z(k)|x(k))\} \\ &= E\{(\nabla_{x(k)} \ln\ p(Z(k)|x(k)))(\nabla_{x(k)} \ln\ p(Z(k)|x(k)))^T\} \end{aligned}$$

We have defined

$$J_Z(m_k) \triangleq E\{(\nabla_{x(k)} \ln\ p(\{z_i(k)\}_{i=1}^{m_k}|x(k))) \times (\nabla_{x(k)} \ln\ p(\{z_i(k)\}_{i=1}^{m_k}|x(k)))^T\}$$

and

$$p(\{z_i(k)\}_{i=1}^{m_k}|x(k)) = \left[\prod_{j=1}^{m_k} p_0(z_j(k))\right] \times \left[(1-\epsilon(m_k)) + \frac{\epsilon(m_k)}{m_k}\sum_{i=1}^{m_k}\frac{p_1(z_i(k))}{p_0(z_i(k))}\right]$$

We get[3], after a small correction[4]:

Theorem

$$J_Z = q_2(P_d, \lambda, R, V) J_Z^0 = q_2(P_d, \lambda, R, V) H^T R^{-1} H$$

and ultimately

Theorem

$$J_{n+1} = Q^{-1} + q_2(P_d, \lambda, R, V) H^T R^{-1} H - Q^{-1} F (J_n + F^T Q^{-1} F)^{-1} F^T Q^{-1}$$

[3] X. Zhang, P. Willett and Y. Bar-Shalom, "A Dynamic Cramer-Rao Lower Bound for Target Tracking in Clutter," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 41, No. 4, pp. 1154-1157, October 2005.

[4] Y. Bar-Shalom, X. Zhang and P. Willett, "A Simplification of the Dynamic Cramer-Rao Bound for Target Tracking in Clutter," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 47, No. 2, pp. 1481-1482, April 2011.

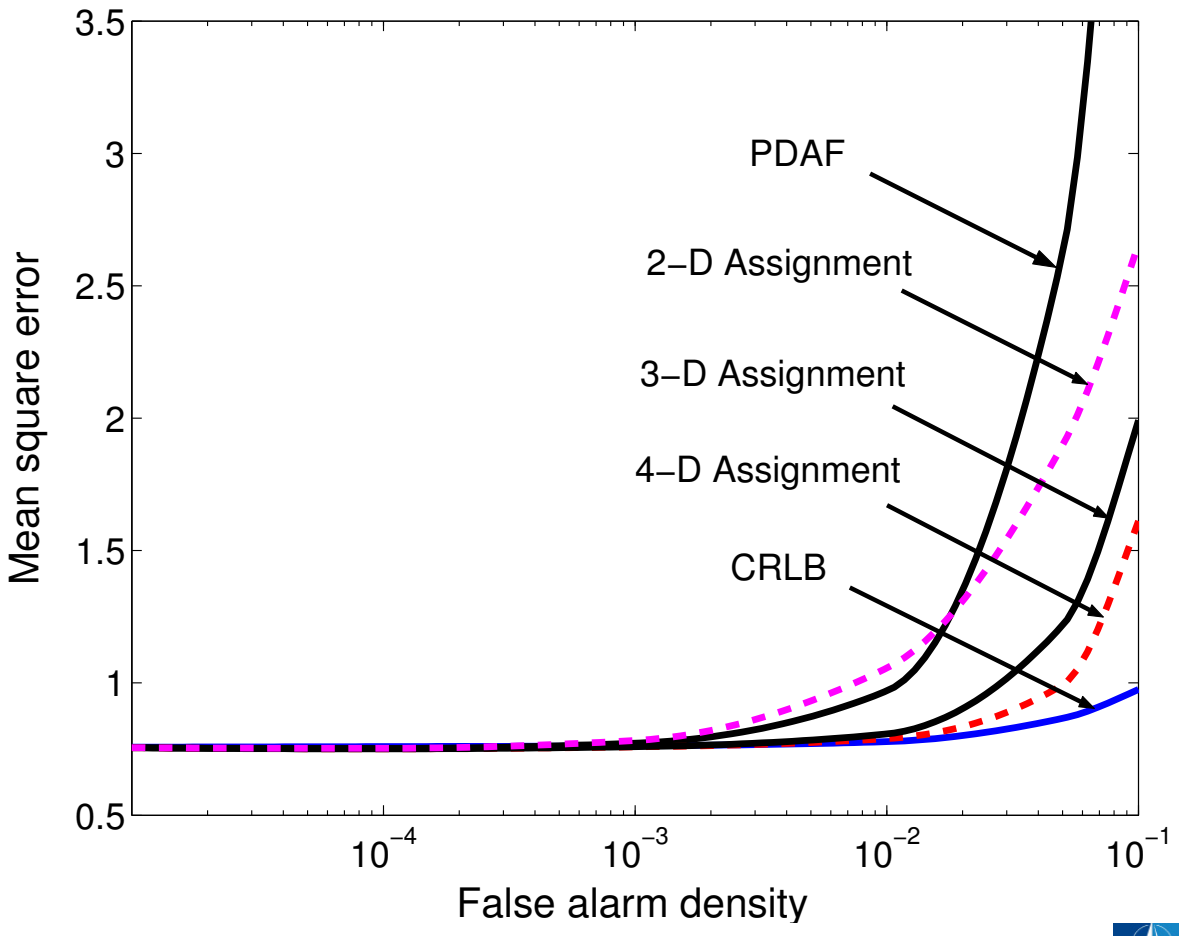
Some IRF's

Value of q_2 at different false alarm densities and probabilities of detection. The state and measurement noises are set as $\tilde{q} = 1$ and $\sigma_w^2 = 1$, and measurements are two-dimensional.

	$P_D = 0.7$	$P_D = 0.8$	$P_D = 0.9$
$\lambda = 0.0001$	0.6814	0.7700	0.8890
$\lambda = 0.0020$	0.6341	0.7472	0.8491
$\lambda = 0.0040$	0.5722	0.6860	0.8144
$\lambda = 0.0060$	0.5460	0.6498	0.7727
$\lambda = 0.0080$	0.5257	0.6290	0.7457
$\lambda = 0.0100$	0.5037	0.6096	0.7367
$\lambda = 0.0120$	0.4679	0.5729	0.6938
$\lambda = 0.0140$	0.4479	0.5343	0.6801
$\lambda = 0.0160$	0.4356	0.5330	0.6590
$\lambda = 0.0180$	0.4180	0.5133	0.6312

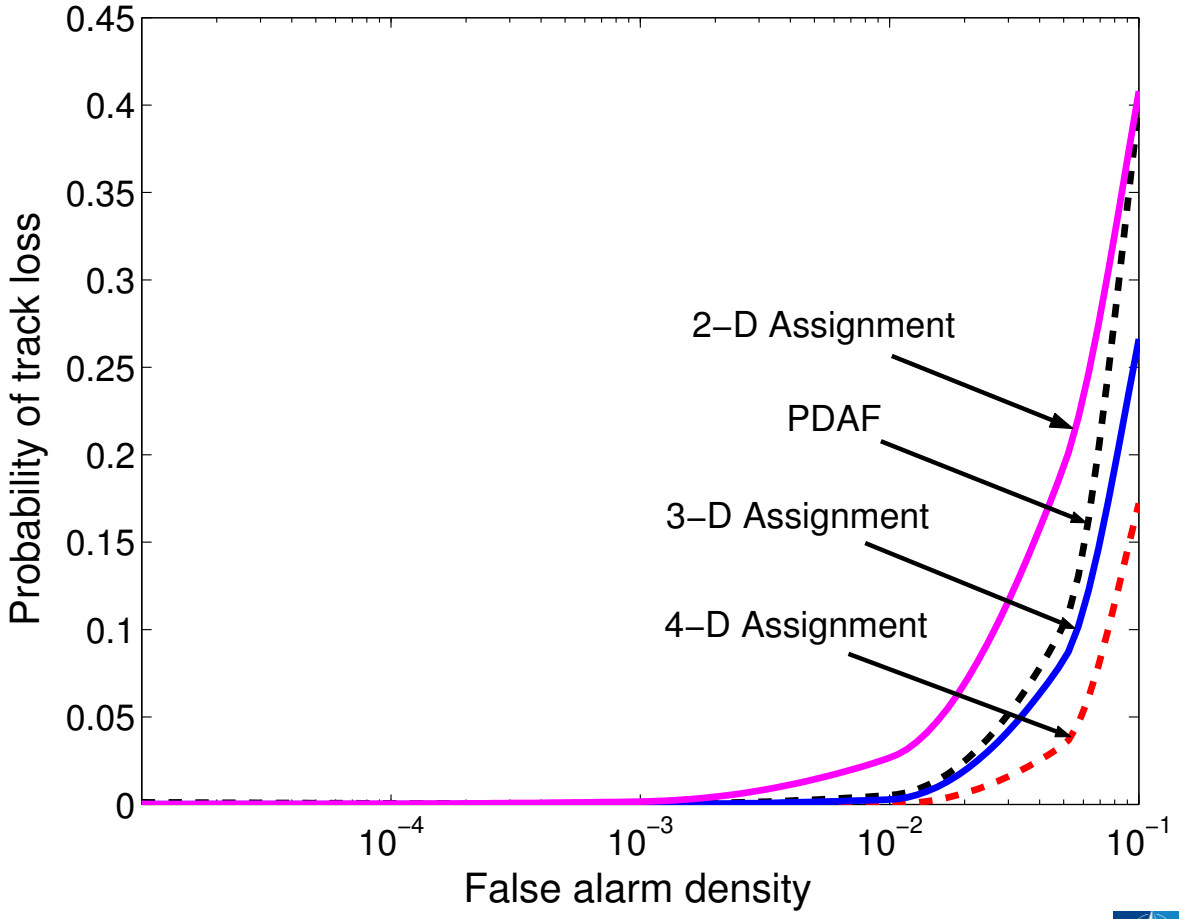
Perfect Detection Example

CRLB compared to MSE of 2-D, 3-D and 4-D assignment and of PDAF, with $P_d = 1$ at different false alarm densities. Only MSE's from those tracks that are not lost are considered.



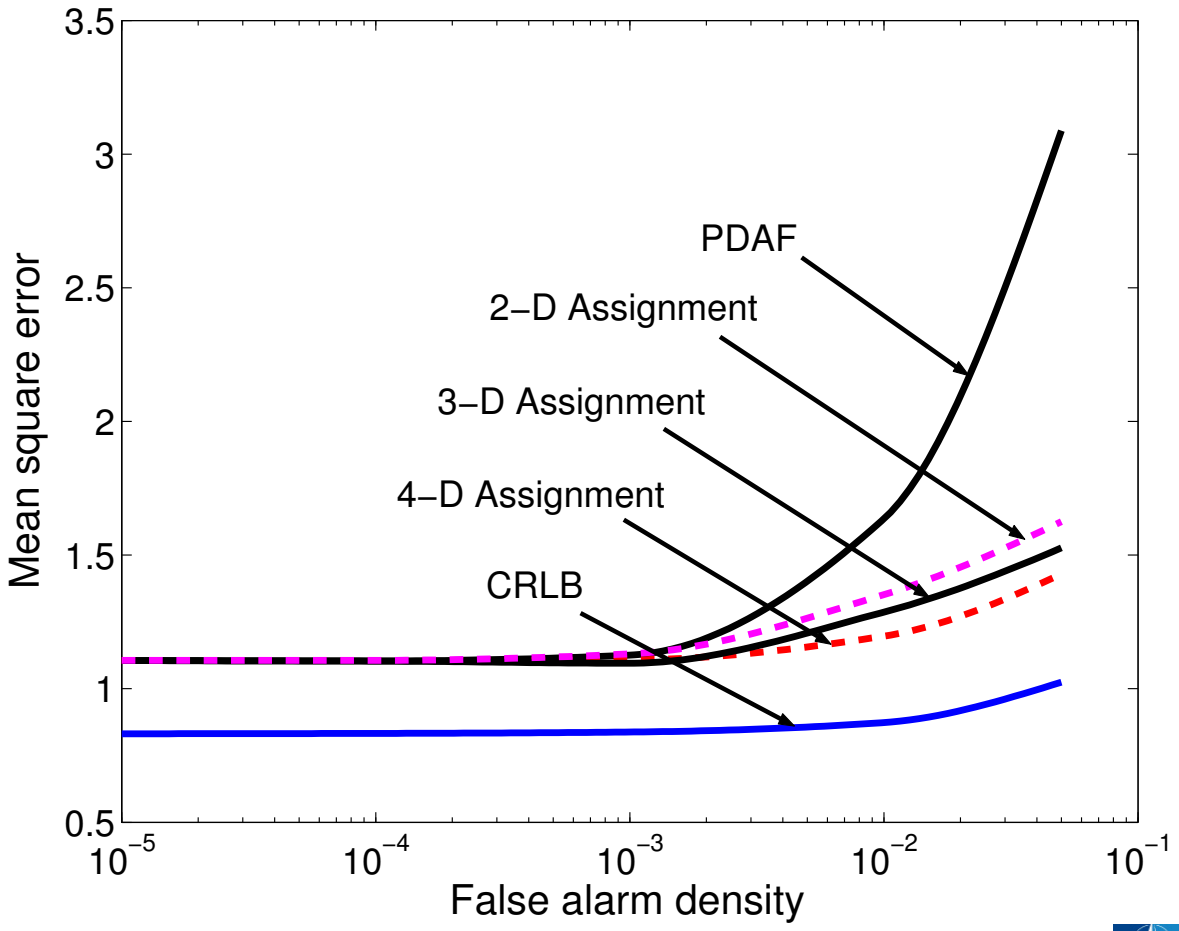
Perfect Detection Example

Probability of track loss, for 2-D, 3-D and 4-D assignment and PDAF, with $P_d = 1$ at different false alarm densities. While in the previous plot it might have seemed that the PDAF was inferior to GNN, in fact the PDAF penalizes itself by retaining more tracks.



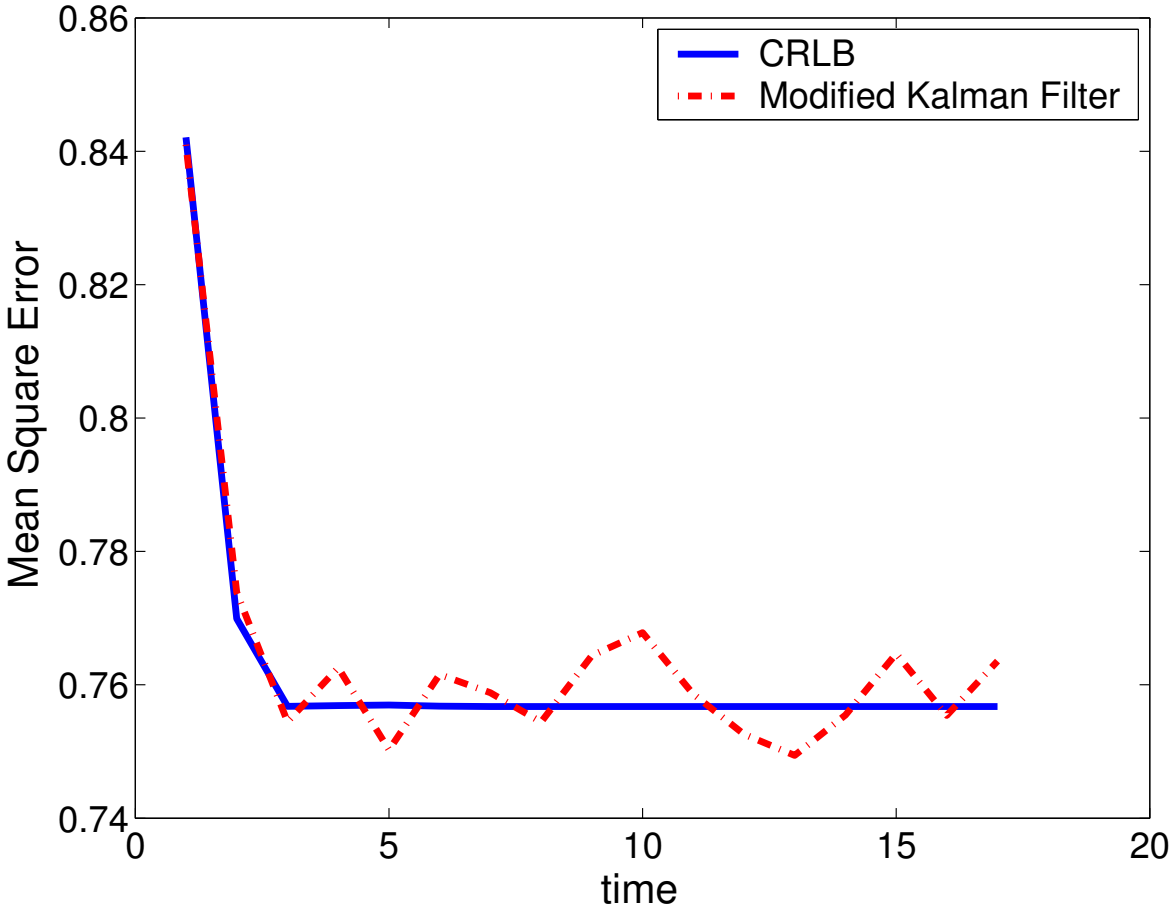
Imperfect Detection Example

CRLB compared to MSE of 2-D, 3-D and 4-D assignment and of PDAF, with $P_d = 0.9$ at different false alarm densities. Only MSE's from those tracks that are not lost are considered.



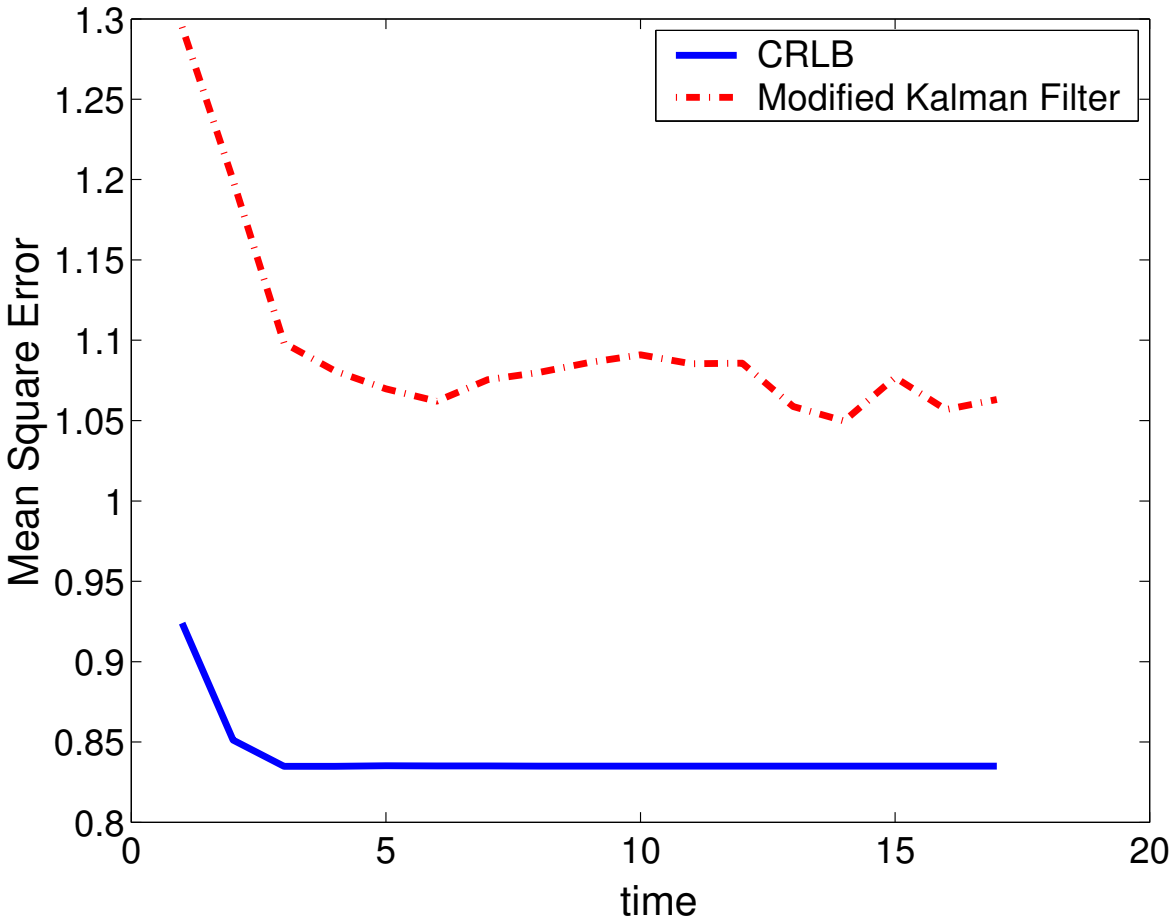
Compare to Kalman Filter

Kalman Filter MSE versus CRLB with $P_d = 1.0$ and no false alarms. As required, the CRLB is tight.



Compare to Kalman Filter With Missed Detections

Kalman Filter MSE versus CRLB with $P_d = 0.9$ and no false alarms. Note that $p(x|Z, m_k = 0)$ is Gaussian, and $p(x|Z, m_k = 1)$ is another Gaussian. But $p(x|Z)$ is a mixture – there is no efficient estimator.



Measurement Sequence Conditioning

Another approach[5] is to use

$$E\{[\hat{x}(z) - x][\hat{x}(z) - x]^T\} = \sum_{\{m_n\}} Pr(\{m_n\}) E\{[\hat{x}(z) - x][\hat{x}(z) - x]^T | \{m_n\}\}$$

and the RHS is the CRLB conditioned on the number of measurements.

[5] M. Hernandez, A. Farina, B. Ristic, "PCRLB for Tracking in Cluttered Environments: Measurement Sequence Conditioning Approach," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 42, no. 2, pp. 680-704, April 2006.

The expectations on the RHS are still of the form

$$J_{n+1} = Q^{-1} + q_2(P_d, \lambda, R, V|m_n)H^T R^{-1} H \\ - Q^{-1} F (J_n + F^T Q^{-1} F)^{-1} F^T Q^{-1}$$

but now q_2 is a single multidimensional integral, not a sum of those. This bound is provably tighter than the Zhang/Willett/Bar-Shalom bound. Practically, the RHS must be evaluated via Monte Carlo, as unless the number of scans is very small the sum cannot be exhaustive. As such, it is a tighter "approximate bound" as opposed to a looser exact bound.

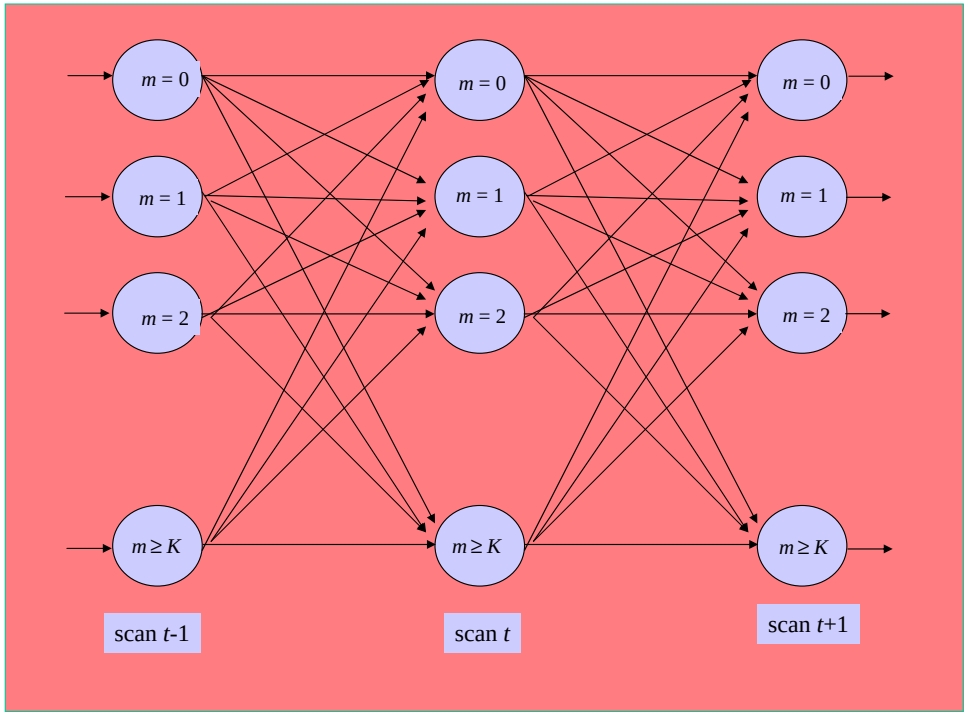
Hybrid Conditional Averaging (HYCA)

The estimation performance of the PDAF can also be evaluated using a Markov chain approach on the number of measurements at each scan.

- X. Li and Y. Bar-Shalom, “Stability Evaluation and Track Life of the PDAF for Tracking in Clutter”, *IEEE Transactions on Automatic Control*, pp. 588-602, May 1991.
- D. Kershaw and R. Evans, “A Contribution to Performance Prediction for Probabilistic Data Association Tracking Filters,” *IEEE Transactions on Aerospace and Electronic Systems*, pp. 1143-1148, July 1996.

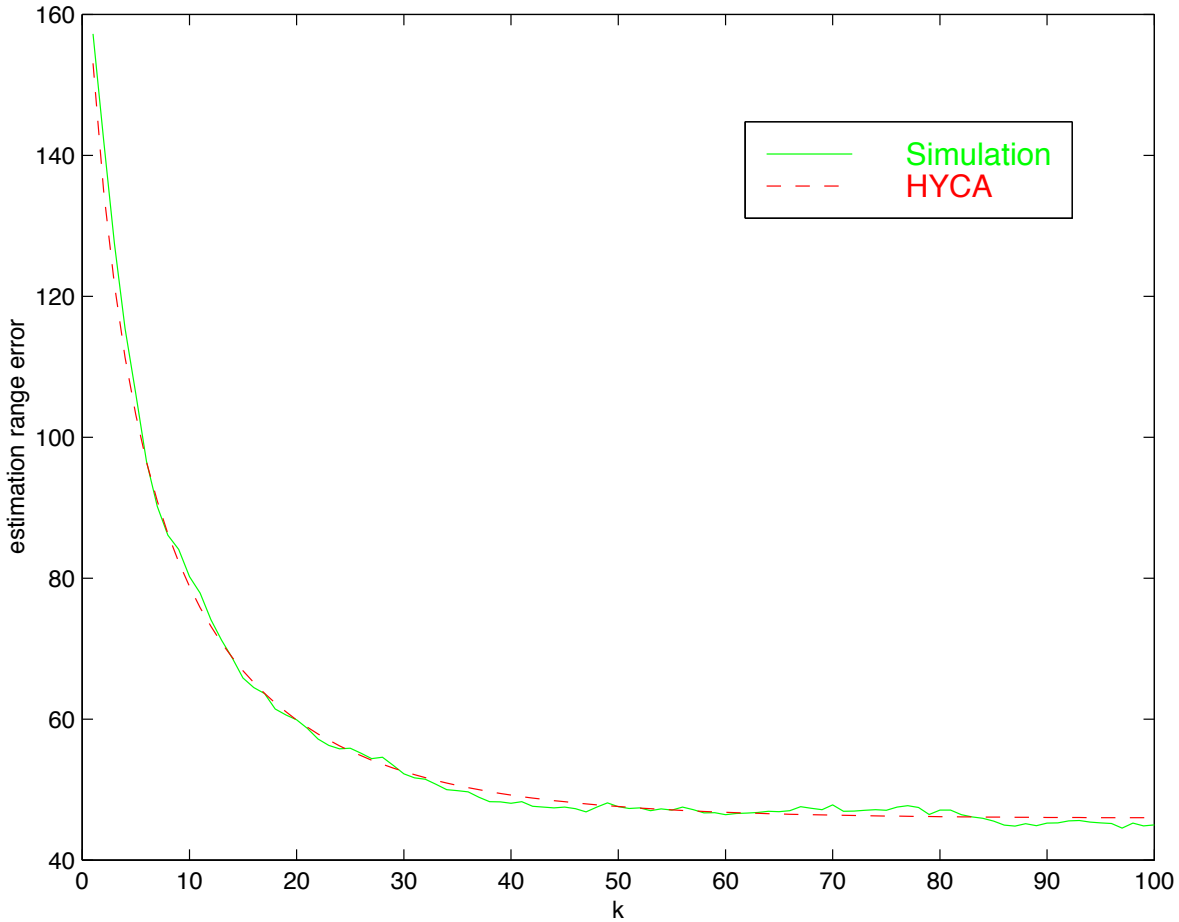
More HYCA

At each scan, and for each branch the PDAF covariance – which would be data-dependent – is in HYCA computed via q_2 (a simplified form, called u_2) or by direct integration. As far as I know HYCA has not been directly compared to the measurement-sequence conditioned BCRLB.



HYCA Verification

The mean square error (MSE) in range via simulation and using HYCA as a function of scan number for a low-maneuvering target, a PDAF in clutter and a CF waveform. The HYCA tool appears to be reliable.



Even more HYCA

- HYCA has been compared to simulation of the PDAF, and has been found to be quite accurate.
- As a track is becoming lost, m_n will increase due to the gate size. Thus a track that is becoming lost will be further "down" the HMM.
- Hence HYCA is perhaps the only performance prediction approach that is able to provide estimates of track loss and track life, in addition to expressions of accuracy.
- Unfortunately HYCA only makes predictions about the PDAF, not an MHT or assignment approach.

Summary of Target Tracking Performance Issues

- CRLBs (parameter estimation) and BCRLBs (random parameters)
- IRF inflates CRLB for MOU case
- the class of problems for which this happens is quite wide
- there is an exact CRLB for nonlinear filtering
- this once again involves the IRF for the MOU case
- other approaches: sequence-conditioned BCRLB and HYCA

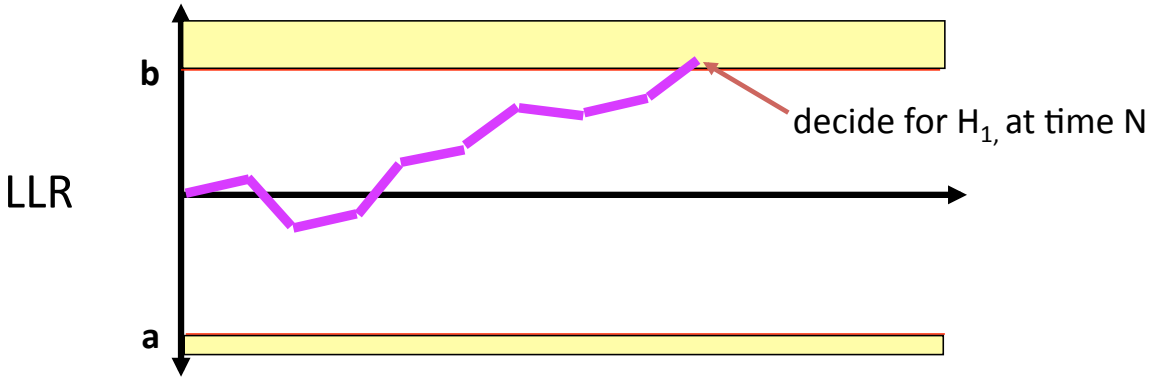
Track Testing Issues

- sequential testing for simple hypotheses
  - the SPRT – threshold setting, performance
  - optimality measures – Wald/Wolfowitz
- quickest detection
  - the Page (cusum) test – iterated SPRT
  - the Shriryaev test – a Bayesian intervenes
- application to tracking
  - track initiation
  - track termination
  - particles
  - fused track management for multistatic sonar

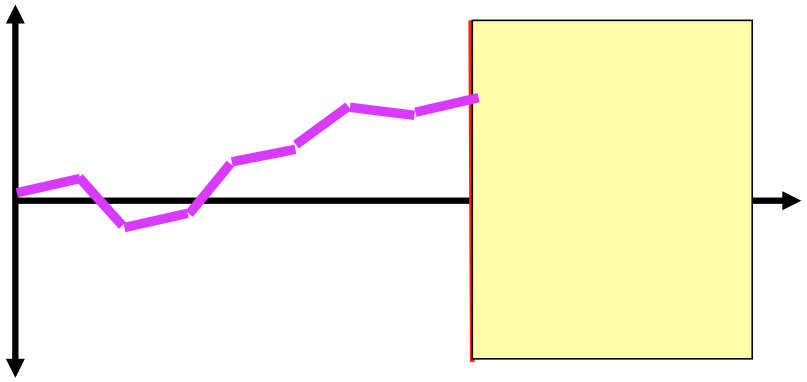
Sequential Testing

Simple binary hypothesis test

- H_0 versus H_1
- T_n=$\log(f_1(x_n)/f_0(x_n))$ is LLR
- stopping rule and decision controlled by upper and lower thresholds

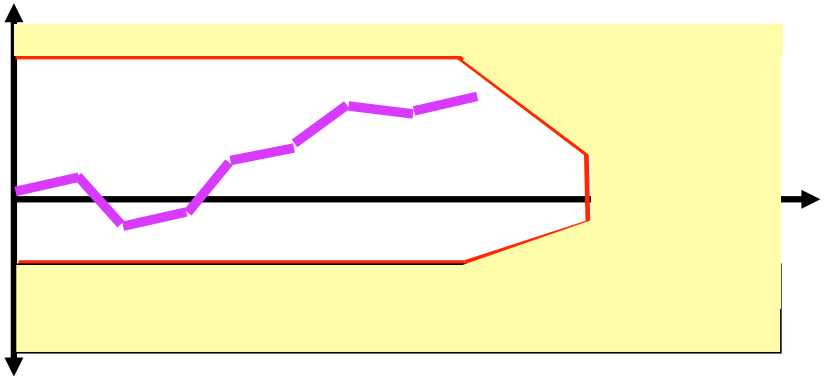


Sequential Testing



fixed sample size (FSS) test

more general test



Sequential Testing: Optimality

- Wald-Wolfowitz Theorem:
  - any test with better than a given α and β performance must have $E(N|H_j)$ performance no better than the corresponding SPRT
  - no point in trying to play games with decision regions or boundaries
- optimality is slippery
  - FSS test
    - has best error performance & bounded N
    - but worse ASN than SPRT
  - truncated SPRT (Tantaratana & Poor)
    - essentially same ASN as SPRT
    - better ASN than FSS test
- multiple hypotheses:
  - a lot is still open
  - Veeravalli, Tartakovsky

Sequential Testing: Analysis

- define $Q_n = \{x^n : \text{ decide for } H_1 \text{ at time } N = n\}$
- define $R_n = \{x^n : \text{ decide for } H_0 \text{ at time } N = n\}$

$$\begin{aligned} \alpha &= \textstyle\sum_n \int_{Q_n} f_0(x^n) \\ &= \tfrac{1}{B} \textstyle\sum_n \int_{Q_n} f_1(x^n) \\ &= \tfrac{\beta}{B} \end{aligned} \qquad \begin{aligned} 1-\beta &= \textstyle\sum_n \int_{R_n} f_1(x^n) \\ &= A \textstyle\sum_n \int_{R_n} f_0(x^n) \\ &= A(1-\alpha) \end{aligned}$$

- Hence Wald's approximations for LR thresholds A & B:
  - $e^b = B = \beta/\alpha$ and $e^a = A = (1-\beta)/(1-\alpha)$
- easier than fixed sample-size (FSS) testing
- Wald's identity, that $E\left\{e^{tT_N}\left[M(t)\right]^{-N}\right\} = 1$, can find average sample numbers (ASNs) for iid case
  - $E(N|H_j) = E(T_N|H_j)/\mu_j$ where μ_j is mean of update under hypothesis H_j
  - also easy to figure out
  - Martingale proof, or using moment generating function

Sequential Testing: Track Initiation

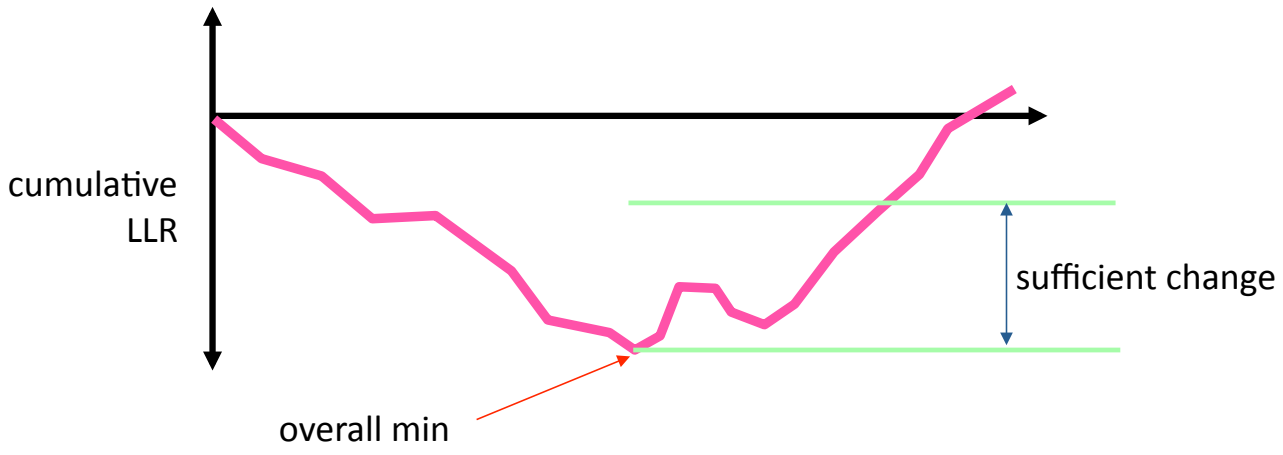
- MHT-style tracking:
  - useful to test tentative tracks, since the starting point of the test is known – as opposed to track termination

$$LLR = \begin{cases} -\frac{1}{2}NIS + \log\left(\frac{V}{\sqrt{|2\pi S|}}\right) + \log\left(\frac{P_d}{P_f}\right) & \text{gated measurement} \\ \log\left(\frac{P_d}{P_f}\right) & \text{no gated measurement} \end{cases}$$

  - from detection-only data use only the P_d & P_f terms
    - this would become a m-out-of-n test
    - Coraluppi would use this anyway
- PDAF, PMHT, etc.:
  - need to modify the LLR
  - the MHT is "easier" since hypothesis is that association is correct

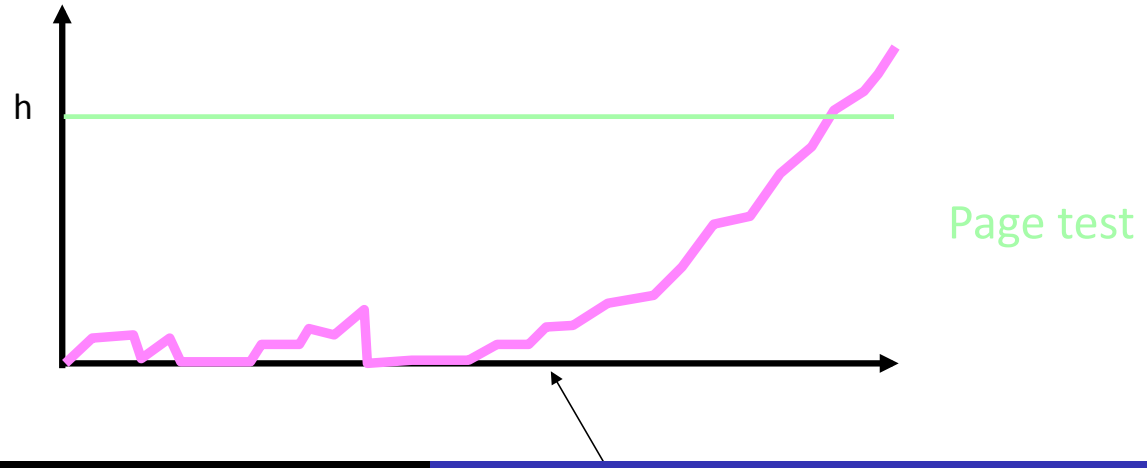
Quickest Detection

- nature changes from H_0 to H_1
  - want quickest alert of this change
  - assume independent data
- intuition:
  - form cumulative LLR
  - if it jumps enough, declare a change



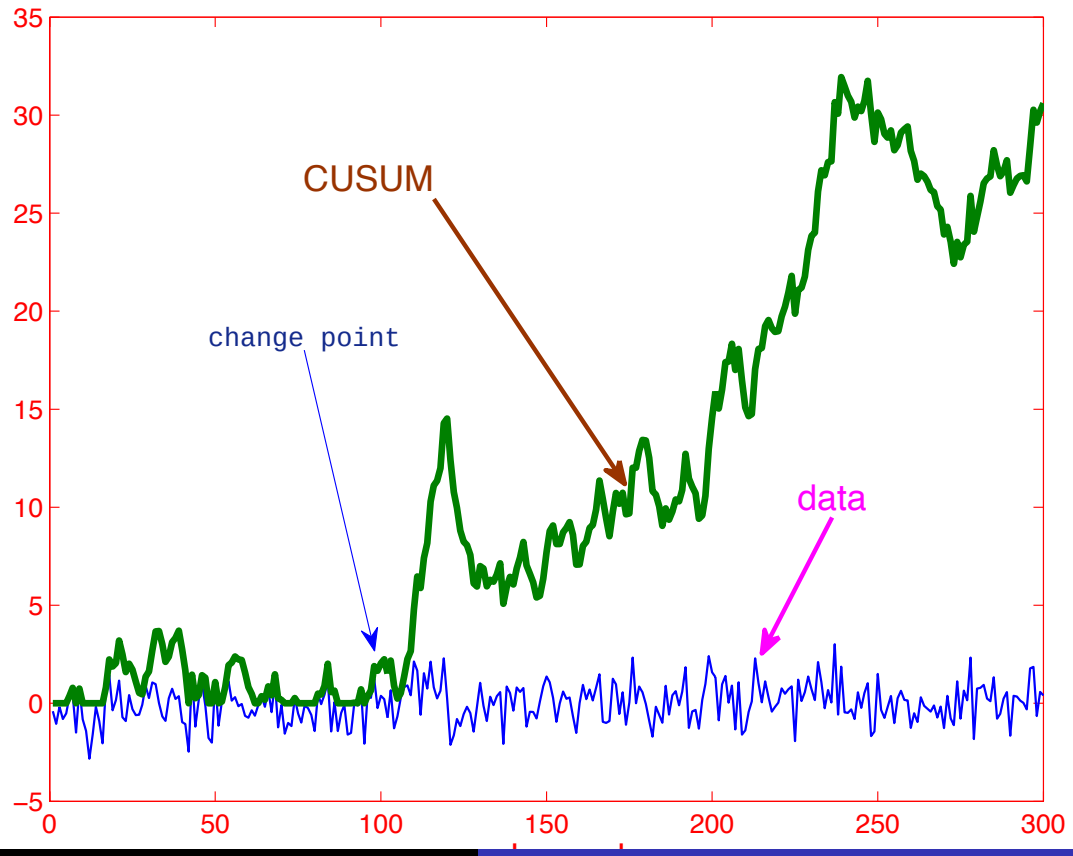
Quickest Detection: CUSUM

- previous sketch is the same as CUSUM (cumulative sum)
  - $T_n = \max\{0, T_{n-1} + g(x_n)\}$
  - iterated SPRT
  - unit Gaussian shift-in-mean:
    $T_n = \max\{0, T_{n-1} + (x_n - \mu/2)\}$ – the "bias" $\left(-\frac{1}{2}\mu\right)$ is very important, since it ensures a downward drift before a change takes place



Quickest Detection: Example

- unit Gaussian with mean ± 0.2 respectively before and after the changepoint



Quickest Detection: Analysis

- there is some nice analysis that comes out of recognizing that a Page test "alert" is actually a sequence of sequential tests – a geometrically-distributed number that end at a ($\rightarrow 0$) and a final one that ends at b (here we call it h)
- define t_i to be the nonzero "root" of the moment generating function identity of the update g under hypothesis i:

$$\int_{-\infty}^{\infty} e^{gt_i} f_i(g) dg \quad = \quad 1$$

- easy to show, given the test is *closed*, that $t_1 < 0 < t_0$
- we have

$$\bar{T} \simeq \frac{1 + ht_0 - e^{ht_0}}{t_0 \int_{-\infty}^{\infty} g f_0(g) dg} \qquad \bar{D} \simeq \frac{1 + ht_1 - e^{ht_1}}{t_1 \int_{-\infty}^{\infty} g f_1(g) dg}$$

Quickest Detection: Analysis

- asymptotically (as h gets large) have $T = e^{\eta D}$, where η is "Page's efficiency"
  - suppose T is 1,000 and D is 10,
  - then can have T of 1,000,000 with a D of 20!
- otherwise, for performance:
  - Siegmund's correction terms
  - Brownian motion or discrete random walk results
  - "C-matrix" or iterated FFT
- optimality:
  - Page procedure is "quickest" for iid case (Lorden, Moustakides)
  - for non-iid case it is known to be quickest for only very special cases where updates are conditionally iid
    - some Markov chains (Moustakides)
    - some special HMMs (Fuh)
  - in many dependent cases Page is wildly suboptimal

Quickest Detection: Track Termination

- applicable to track deletion in MHT
  - continually test for a change in statistics
  - LLRs basically the same as for SPRT (Mahalonobis, due to independence of innovations)
- in nonlinear filtering situations need

$$T_n = \max\left\{0, \log\left(\frac{f(z_{n_0}^n|H_1)}{f(z_{n_0}^n|H_0)}\right)\right\}$$

  where n_0 is the last "reset"
- in general this is difficult, but with a particle filter Boers & Driessen showed a useful result . . .

Quickest Detection: Detection of a Target Spawn

- quickest detection of statistical change from one hypothesis to another – here this might be $J = 1$ target vs. $J + 1 = 2$ unresolved ones
- stopping rule ($N = n$) and update rules (of CUSUM S_n):

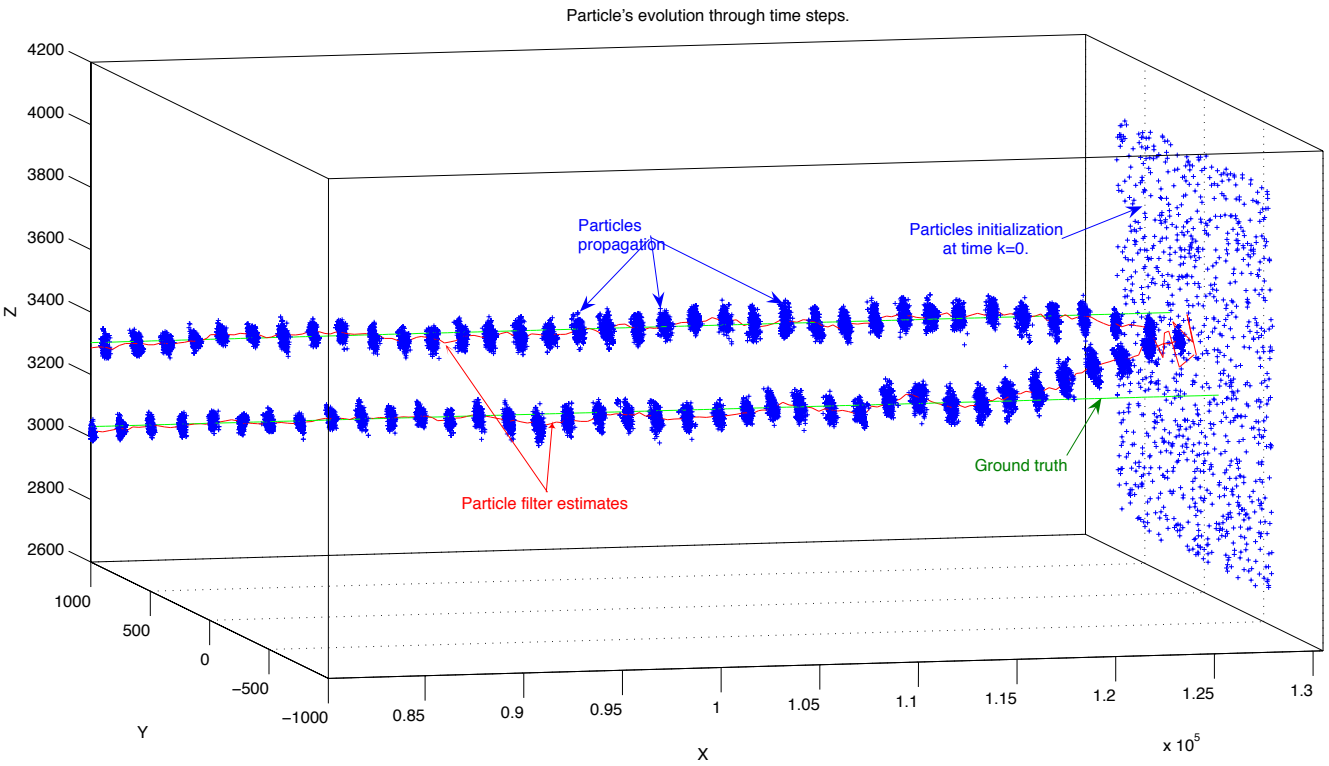
$$N = \arg\min_n \{S_n \geq h\} \text{ and } S_n = \max\{0, S_{n-1} + g(z_n)\}$$

- update is especially simple for particle filter, despite dependence with unresolved monopulse observations z_n:

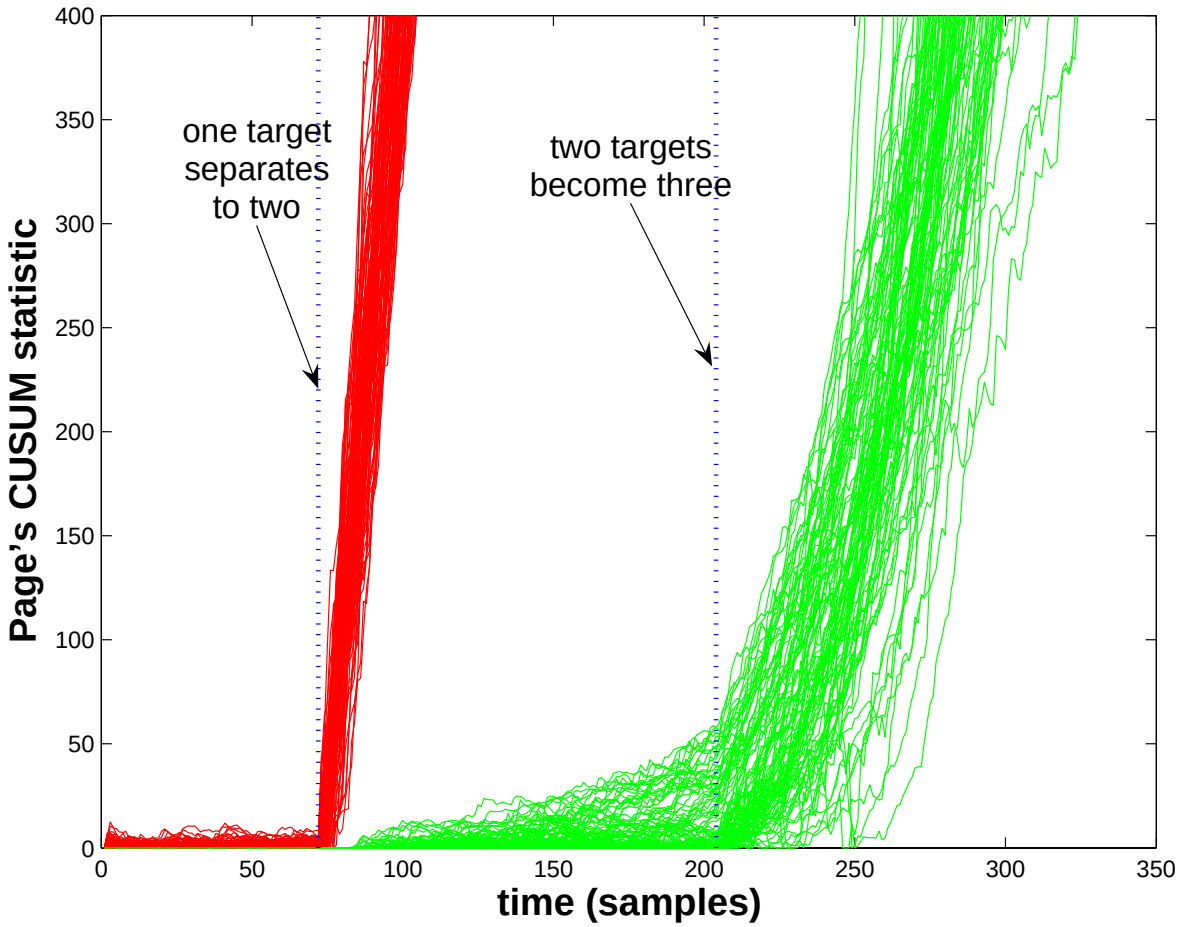
$$g(z_n) = \log\left(\frac{f^{J+1}(z_n)}{f^J(z_n)}\right) = \log\left(\frac{\sum_i \tilde{w}_i^{J+1}}{\sum_i \tilde{w}_i^J}\right)$$

where $\tilde{w}_i^K$ is the unnormalized weight of the ith particle under model K

Quickest Detection: Example of Tracking Two Unresolved Monopulse Targets



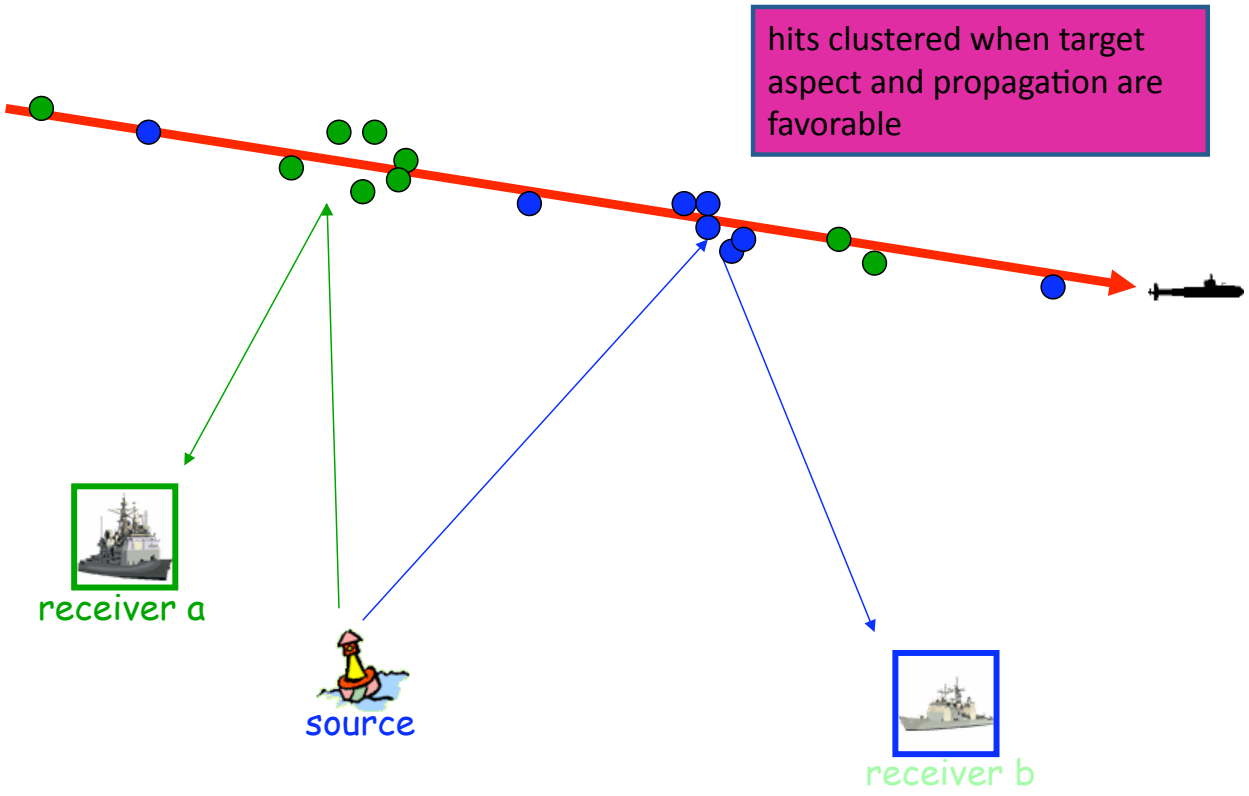
Quickest Detection: Example of Tracking Two Unresolved Monopulse Targets



Quickest HMM Change Detection: Motivation

- multistatic (sonar) targets exhibit fading detection performance
  - periods of high and low detection probabilities (for each pair)
  - caused by geometric, environmental, and geographic effects
- Coraluppi showed that a decentralized track confirmation architecture outperformed a centralized architecture
  - sensor P_d based on a Hidden Markov Model (high/low P_d states)
  - results contradict the axiom that centralized architectures work best
- using a multistatic tracking environment,
  - develop an improved track termination procedure based on change detection/optimal stopping rules

Quickest HMM Change Detection



Quickest HMM Change Detection: Page-Inspired

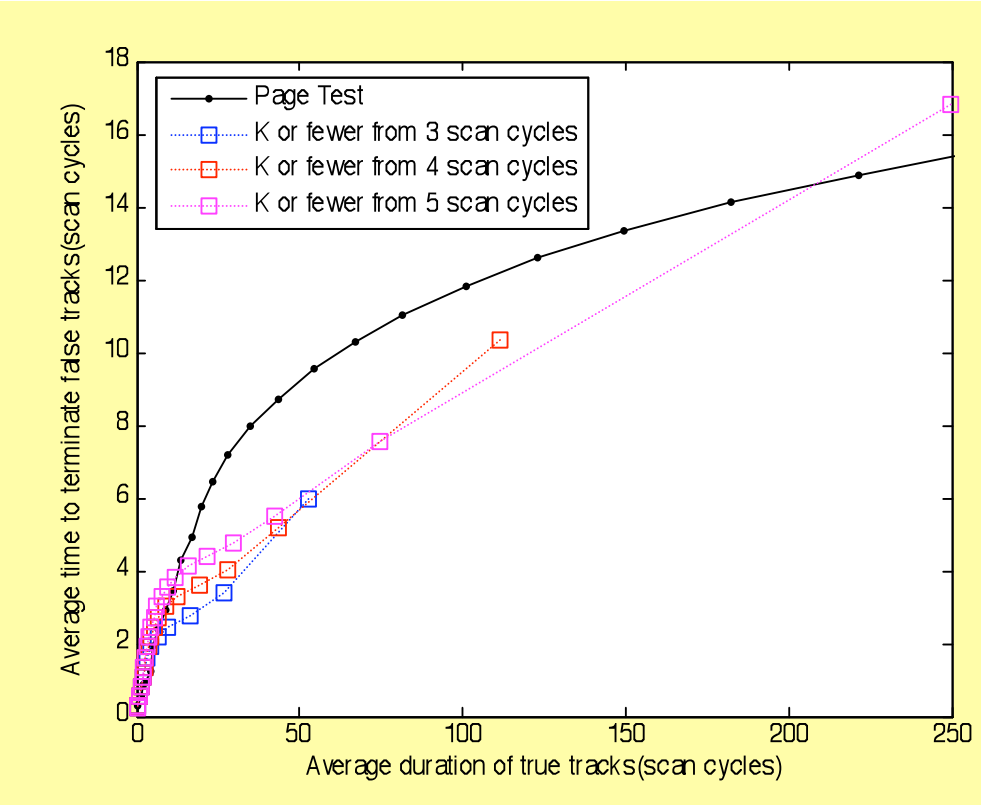
- goal: quickest detection of change in measurement distribution from a true track (H_1) to a false track (H_0)
- Page test
  - proven global optimality for i.i.d. case and some Markov models
  - some recent asymptotic optimality results for HMMs [Fuh 2003], not applicable for this case
  - a sequential test that minimizes delay in detection of a distribution change at a given false alarm

$$T_n = \max\{0, T_{n-1} + g_n\} \text{ where } g_n = \log\left(\frac{Pr(\delta_n|\delta_1^{n-1}, H_0)}{Pr(\delta_n|\delta_1^{n-1}, H_1)}\right)$$

where δ is the detection process

Quickest HMM Change Detection: Page-Inspired

- shown: track termination performance (4 sensors)
- surprising result: Page test is not globally optimal
- LLR innovations are not i.i.d.



Quickest HMM Change Detection: Shiryaev Test

- optimum quickest detection when the problem is formulated using a Bayesian approach (i.i.d. case) a priori probability of change time k_c:

$$\Pr\{k_c=k\}=\begin{cases}\beta_0 & k_c=0\\ (1-\beta_0)\rho(1-\rho)^{k-1} & k_c>0\end{cases}$$

- Applying Bayes rule, the *a posteriori* probability of a change at time $k>0$ is

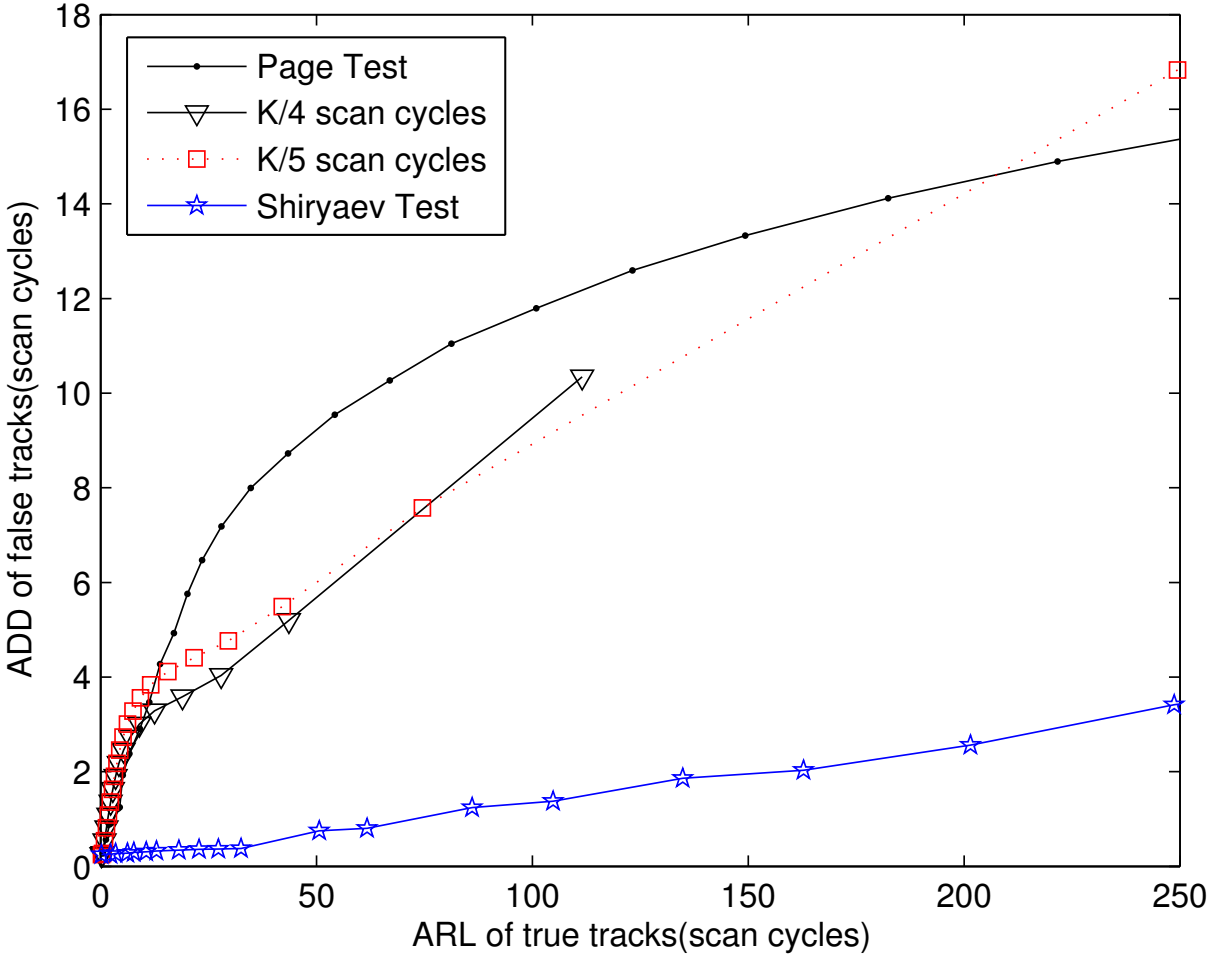
$$\beta_k=\frac{[\beta_{k-1}+(1-\beta_{k-1})\rho]f_0(\delta_k|\delta_1^{k-1})}{[\beta_{k-1}+(1-\beta_{k-1})\rho]f_0(\delta_k|\delta_1^{k-1})+(1-\beta_{k-1})(1-\rho)f_1(\delta_k|\delta_1^{k-1})}$$

- The Shiryaev stopping rule becomes

$$g_k=\ln\frac{\beta_k}{1-\beta_k}=\ln(\rho+e^{g_{k-1}})-\ln(1-\rho)+\ln\frac{f_0(\delta_k|\delta_1^{k-1})}{f_1(\delta_k|\delta_1^{k-1})}$$

where with each scan g_k is compared to a threshold h

Quickest HMM Change Detection: Shiryaev Test



Track Testing Summary

- sequential testing
  - SPRT is "optimal"
  - nice expressions for T, D and thresholds
  - useful for track initiation
  - m-out-of-n rule if detection-only
- quickest detection
  - CUSUM (Page) is "optimal" under independence
  - T is exponential in D, approximate expressions
  - useful for track termination
  - if situation is nonlinear may need particle weights
  - if not independent may need Shiryaev test

Summary Summary

- Lots of small results.
- I'm done.
- Hope this was useful!